

Rough Set Approach to Domain Knowledge Approximation

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Abstract. Classification systems working on large feature spaces, despite extensive learning, often perform poorly on a group of atypical samples. The problem can be dealt with by incorporating domain knowledge about samples being recognized into the learning process. We present a method that allows to perform this task using a rough approximation framework. We show how human expert's domain knowledge expressed in natural language can be approximately translated by a machine learning recognition system. We present in details how the method performs on a system recognizing handwritten digits from a large digit database. Our approach is an extension of ideas developed in the rough mereology theory.

Keywords: Rough mereology, concept approximation, domain knowledge, machine learning, handwritten digit recognition.

1. Introduction

Several decades of research on handwritten digit recognition have yielded significant success, with many efficient algorithms and recognition systems that provide impressive results. Extensive experiments however have shown that no matter how efficient and sophisticated the techniques employed are, a considerable set of samples remain difficult to deal with. This problem is subject to a recently growing trend in the community which stresses on the special treatment of difficult areas in the feature space.

We present a scheme for incorporating domain knowledge about handwritten digit samples into the learning process. The knowledge is provided by a hypothetical expert that will interact with the classification system during a later phase of the learning process, providing certain “guidance” to the difficult

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task of adaptive searching for correct classifiers. The main underlying assumption is that when the feature space is as large as in the case of handwritten digits, algorithms seeking to approximate human reasoning functions will perform better if equipped with domain knowledge provided by a human expert.

In distinction to most popular domain knowledge based approaches widely used in recognition systems, ours concentrates on specific difficult, error-prone samples encountered during the learning phase. The expert will pass the correct classification of such cases to the system along with his explanation on how he made the decision on the class identity of the sample. The system then incorporates this knowledge, using its own descriptive language and primitives, to rebuild its classifiers.

In this paper, we describe the expert knowledge representation scheme, based on the rough mereology approach to concept's approximation, as well as the mechanism of interaction between expert and the classifier construction system presented in [1]. It is easy to observe that the approach is not limited to handwritten digits, but can be readily used in classifying other structured objects.

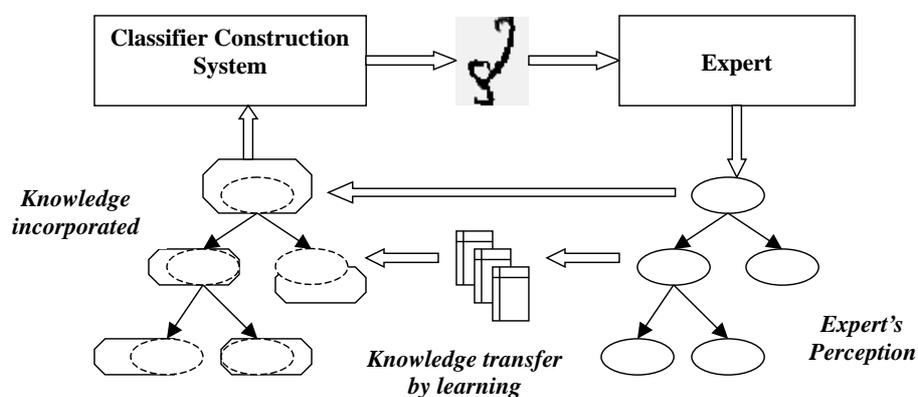


Figure 1. General Outline

2. Adaptive recognition system

The development of OCR in general and handwritten digit recognition in particular over the years yielded many highly effective description models for the analysis of digit images. For the research in this paper we have chosen the Enhanced Loci coding scheme, which assigns to every image pixel a code reflecting the topology of its neighborhood. Each pixel in a digit image is scanned to determine if it is close to a black stroke and if so, what is the relative position of the stroke: To the North, East, South or the West of the pixel. The pixel then is assigned a code that reflects its particular position related to its black neighborhood. Codes for neighboring regions then can be combined to reflect the internal structure of the digit image (See Fig. 2).

The Enhanced Loci algorithm, though simple, has proved to be very successful in digit recognition. For a detailed description of the Loci coding scheme, see [3].

Once the Loci coding is done, the digit image is segmented into regions consisting of pixels with the same code value. These regions then serve as *primitives* to build the graph representation of the image.

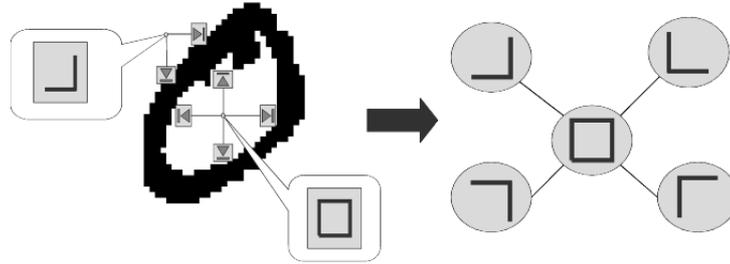


Figure 2. Graph Model based on Loci Encoding.

3. "Hard" samples detection

Typical digit recognition system classifies an unknown sample by computing its "distance" or "similarity" to a collection of prototypes established during the training phase. The most popular, the k -nearest neighbor method assigns the sample to the class represented by the majority of its k neighbor samples. This and other traditional methods perform this task on a uniform basis irrespective of the "difficulty" of the investigated sample, whereas it is obvious that not all samples are equally easy to classify. Samples that are far from the "centers" of the class prototypes tend to fall on the boundaries between classes, are more error-prone and hence can be regarded as more "difficult". While existing recognition algorithms perform well on most "regular" digit subclasses, the majority of "difficult" samples can only be recognized with additional knowledge, most often provided by a human expert.

A straightforward criteria to detect such samples can be defined as follows:

Let PG_k be the prototype graph set constructed for class k during the training phase and d_k be the distance function established for that class. An unknown digit sample u_k of class k is considered "difficult", "hard" or "atypical" if:

$$d_k(u_k, PG_k) \geq \rho \max\{d(v, PG_k) : v \in TR \wedge CLASS(v) = k\}$$

where $\rho \in (0, 1]$ is some cut-off threshold and TR is the training table.

Alternatively, samples repeatedly misclassified during cross-validation tests in the training phase can as well be considered "difficult". Since class identities of all digit samples are known during the training phase, we can detect the "hard" ones beforehand and submit them to the expert for review.

4. Passing domain knowledge to classifiers

Now suppose that at some point during the training phase, we detect a number of samples of class k that are misclassified. The samples are submitted to the expert, which returns not only the correct class identity, but also an explanation on *why*, and perhaps more importantly, *how* he arrived at his decision.

4.1. General scheme

We will assume that the expert explanation is expressed as a rule:

$$[CLASS(u) = k] \equiv \mathfrak{S}(EFeature_1(u), \dots, EFeature_n(u))$$

where $EFeature_i$ represents the expert’s perception of some characteristics of the sample u , while synthesis operator \mathfrak{S} represents his perception of some relations between these characteristics.

It is assumed that the actual structure of $EFeature_i$ might not be flat, but can be multi-layered with various sub-concepts at subsequent levels of abstractions.

For example, the expert may express his perception of digit ‘5’ as (See Fig.3):

$$[CLASS(u) = '5'] \equiv a, b, c, d \text{ are parts of } u; \text{ "Above_Right"(}a,b\text{); "HStroke"(}a\text{); } b = Compose(c, d); \\ \text{"VStroke"(}c\text{); "WBelly"(}d\text{); "Above"(}c, d\text{),}$$

where $Compose$ is an assembling operator that produces a bigger part from smaller components.

The above means if there is a west-open belly below a vertical stroke and the two have a horizontal stroke above-right in the sample’s image, then the sample is a ‘5’.

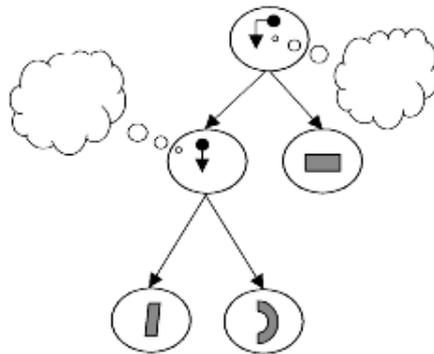


Figure 3. Object Perception Provided by Experts

The main challenge here is that the expert explanation is expressed in his own descriptive language, intrinsically related to his natural perception of images and often heavily based on natural language constructs (a *foreign language* L_f), while classifiers have a different language designed to, for example, facilitate the computation of physical characteristics of the images (a *domestic language* L_d). For example, the expert may view sample images as a collection of shapes or strokes (“A ‘6’ is something that has a neck connected with a circular belly”) while the recognition system regards the samples as graphs of Loci-based nodes. The knowledge passing process hence can be considered as approximating of expert’s concept by the classifier construction system.

It is essential here that the concept matching should not be “crisp”, but expressed by some rough inclusion measures, determining if something is satisfying the concept to a certain degree [1]. For instance, a stroke at 85 degree to the horizontal can still be regarded as a vertical stroke, though obviously with a degree less than 1.0. The extent of such variations may be provided by the expert (e.g., by providing samples that represent “extreme” instances of a given concept).

Let us further assume that such an inclusion measure is denoted by $Match(p, C) \in [0, 1]$, where p is a pattern (or a set of patterns) encoded in L_d and C is a concept expressed in L_f . An example of a

concept inclusion measure would be:

$$Match(p, C) = \frac{|\{u \in T : Found(p, u) \wedge Fit(C, u)\}|}{|\{u \in T : Fit(C, u)\}|}$$

where T is a common set of samples used by both the system and the expert to communicate with each other on the nature of expert's concepts, $Found(p, u)$ means a pattern p is present in u and $Fit(C, u)$ means u is regarded by the expert as fit to his concept C .

Our principal goal is, for each expert explanation, to find sets of patterns Pat, Pat_1, \dots, Pat_n , and a relation \mathfrak{S}_d such that

$$\begin{aligned} &\mathbf{if} (\forall i : Match(Pat_i, EFeature_i) \geq p_i) \wedge (Pat = \mathfrak{S}_d(Pat_1, \dots, Pat_n)) \\ &\quad \mathbf{then} Quality(Pat) > \alpha \end{aligned}$$

where $p, p_i : i \in \{1, \dots, n\}$ and α are certain cutoff thresholds, while the *Quality* measure, intended to verify if the target pattern Pat fits into the expert's concept of digit class k , can be any, or combination, of the following criteria:

- $Support_{CLASS=k}(Pat) = |\{u \in U : Found(Pat, u) \wedge CLASS(u) = k\}|$
- $Match_{CLASS=k}(Pat) = \frac{Support(Pat)}{|\{u \in U : Found(Pat, u)\}|}$
- $Coverage_{CLASS=k}(Pat) = \frac{Support(Pat)}{|\{u \in U : CLASS(u) = k\}|}$ where U is the training set.

In other words, we seek to translate the expert knowledge into the domestic language as to generalize the expert reasoning to the largest possible number of physical digit samples. The requirements on inclusion degrees ensure the stability of the target reasoning scheme, as the target pattern Pat retains its quality regardless of deviations at input patterns Pat_i as long as they still approximate the expert concept $EFeature_i$ to degrees at least p_i . This may also be described as *pattern robustness*.

Another important aspect of this process is its *concept approximation robustness*, meaning not only that it does ensure that the target pattern Pat will retain its quality with regard to input patterns deviations in inclusion degrees, but it also should guarantee that if we have some input patterns Pat_i equally "close" or "similar" to $EFeature_i$, then the target pattern $Pat' = \mathfrak{S}_d(Pat'_1, \dots, Pat'_n)$ will meet the same quality requirements as Pat to a satisfactory degree. This leads to an approximation of $EFeature_i$ that is independent of particular patterns Pat_i , allowing us to construct approximation schemes that focus on inclusion degrees p_i rather than on a specific input patterns Pat_i .

4.2. Pattern approximation

One can observe that the main problem that poses itself here is how to establish the interaction between the expert who reasons in L_f and the classifier construction system that uses L_d . Here, once again, the system has to learn (with the expert help) "what he meant when he said what he said." More precisely, the system will have to construct the measure *Match* and the relation \mathfrak{S}_d .

In order to learn the measure *Match*, what essentially means that we are trying to learn the expert concept of $EFeature_i$, we ask the expert to examine a given set of samples U and provide a decision

table (U, d) where d is the expert decision whether $EFeature_i$ is present in a particular sample from U , for instance, whether a sample has a "WBelly" or not. We then try to select a set of features in the system domestic language that will approximate the decision d , for example, number of pixels with the NES Loci code. For example:

	WBelly
u_1	yes
u_2	no
...	...
u_n	yes

 \Rightarrow

	#NES	WBelly
u_1	252	yes
u_2	4	no
...
u_n	90	yes

Table 1. Pattern approximation

In the above table, #NES is the number of white pixels that are bounded in all directions except to the West.

It is assumed that the set U will not be too large to ensure the feasibility of acquiring the expert answers that facilitates this feature selection task. Experiments have shown that for popular features such as the presence of a circle or a straight stroke, sometimes it is enough to employ some simple greedy heuristics. For more complex patterns, one can use some efficient evolutionary strategies.

4.3. Pattern relation approximation

Having approximated the concepts $EFeature_i$, we can try to translate the expert relation \mathfrak{S} into our \mathfrak{S}_d by asking the expert to go through U and provide us with the additional attributes of whether he found the $EFeature_i$ and a decision d if the relation \mathfrak{S} holds.

	VStroke	WBelly	Above		#V_S	#NES	$S_y < B_y$	Above
u_1	yes	yes	yes	u_1	34	252	yes	yes
u_2	yes	no	no \Rightarrow	u_2	45	4	no	no
...
u_n	yes	yes	no	u_n	40	150	no	no

 \Downarrow

	$Match(\#V_S, VStroke)$	$Match(\#NES, WBelly)$	$Match(S_y < B_y)$	$Match(Above)$
u_1	0.85	0.95	(yes, 1.0)	(yes, 0.9)
u_2	0.95	1.0	(no, 0.1)	(no, 0.05)
...
u_n	0.90	0.70	(no, 0.3)	(no, 0.15)

Table 2. Pattern relation approximation

We then replace the attributes corresponding to $EFeature_i$ with the characteristic functions of the domestic feature sets that approximate those concepts and try to add other features, possibly induced

from original domestic primitives, in order to approximate the decision d . Again, this task should be resolved by means of adaptive or evolutionary search strategies without too much computing burden.

Here is an example how the concept of a “vertical stroke” “above” a “west-open belly” would be approximated:

In the above table, #V_S is the number of black pixels having the Loci code characterizing a vertical stroke and $S_y < B_y$ tells whether the median center of the stroke is placed closer to the upper edge of the image than the median center of the belly. The third table shows degrees of inclusion of these domestic features in the original expert’s concept “VStroke”, “WBelly” or “Above” respectively.

A further step of this method is to enhance the pattern matching process with soft degrees of concept matching. Instead of implicit numeric degrees of inclusion, we can ask the expert to provide us with a description on of how he perceives a particular feature on a particular subsets of samples.

The Table 2 now becomes Table 3.

	<i>Match(VStroke)</i>	<i>Match (WBelly)</i>	<i>Match (Above)</i>	<i>Is 5?</i>
u_1	<i>Fair</i>	<i>Strong</i>	<i>Strong</i>	<i>Strong</i>
u_2	<i>Strong</i>	<i>Strong</i>	<i>Weak</i>	<i>Fair</i>
...
u_n	<i>Fair</i>	<i>Weak</i>	<i>Weak</i>	<i>Weak</i>

Table 3. Reasoning with soft matching degrees

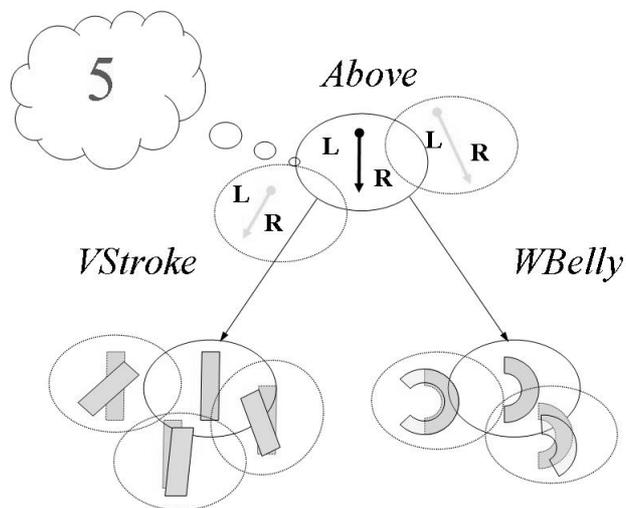


Figure 4. Evolution of Expert’s Perception

The soft degrees *Weak*, *Fair*, *Strong* form a rough idea of how the expert perception of a particular feature concept evolves with the patterns found in sample images. Evaluating how this evolution influence the expert decision on the image features and finally, its class identity, can be crucial to the construction of the researched classifiers.

The main idea behind pattern relation approximation is the observation that while "crisp" learning essentially requires a search through the product space of many $EFeature_i$, which is often complex and computationally prohibitive, the additional relation \mathfrak{S} will provide a "guidance" that helps navigate throughout the possible feature space and ultimately will allow a more successful search result.

It is noteworthy that the concept approximation process should work under a requirement to the quality of the searched global pattern Pat , which should have a substantial support among other samples, not examined by the expert, from the training collection. This will ensure the knowledge passed by the expert on a particular example is actually generalized into more global concept.

5. Classifying unknown samples

Once we have established domestic language feature sets and constrain relations approximating the expert reasoning on a particular type of digits, we essentially obtained a multi-layered reasoning scheme. An unknown sample can then be checked against this scheme to see whether it bears enough characteristic traits of this digit class. This can be done by decomposing the unknown pattern according to the structure of the reasoning scheme, checking its matching degree at each level and subsequent computing its matching degree at higher levels up to the root.

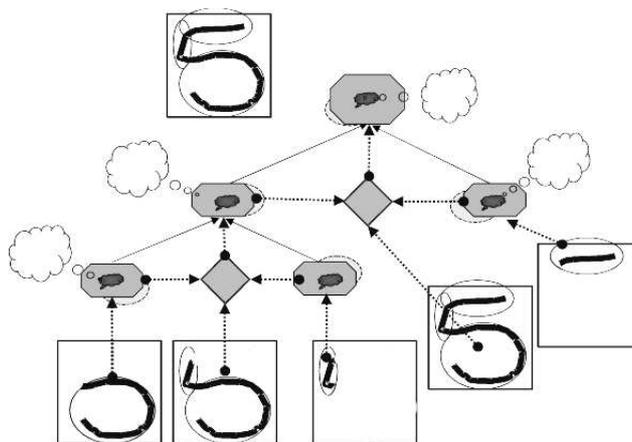


Figure 5. Recognizing new samples

It can be observed that each pattern and constraint relation set at a particular level of reasoning determines a cluster of samples matching it. Based on the similarity measures developed during the training phase that correspond to the pattern sets, we can derive the distance of unknown samples to each cluster and, in consequence, develop the inclusion measure of a new sample in the concept approximated by the cluster.

It is also essential to note that the quality requirement imposed while searching for the target patterns ensure that the obtained classifier is stable, i.e. resistant to certain derivations in the input sample. It is enough for the new input to match the basic patterns at the lowest level Pat_i to a degree greater than the satisfactory threshold p_i , and the outcome classification decision will be guaranteed to at least a satisfactory degree of accuracy.

6. Conclusion

A method for incorporating domain knowledge into the design and development of a classification system is presented. We have demonstrated how approximate reasoning scheme can be used in the process of knowledge transfer from human expert's ontology, often expressed in natural language, into computable pattern features. Developed schemes ensure stability and adaptability of constructed classifiers. We have shown how granular computing, equipped with rough mereology concepts can be effectively applied to a highly practical field such as OCR and handwritten digit recognition. Developed framework might as well be used in classifying other objects including, but not limited to, fingerprints, mugshots, iris scans as well as for more complex tasks like project WITAS (<http://www.ida.liu.se/ext/witas/>).

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