Constrained Sums of Information Systems

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Abstract. We study properties of infomorphisms between information systems. In particular, we interpret infomorphisms between information systems in terms of sums with constraints (constrained sums, for short) that are some operations on information systems. Applications of approximation spaces, used in rough set theory, to study properties of infomorphisms are included.

1 Introduction

One of the main task in granular computing is to develop calculi of information granules [7], [13], [9], [10]. Information systems used in rough set theory are particular kinds of information granules. In the paper we introduce and study operations on such information granules basic for reasoning in distributed systems of information granules. The operations are called constrained sums. They are developed by interpreting infomorphisms between classifications [1]. In [11] we have shown that classifications [1] and information systems [5] are, in a sense, equivalent. We also extend the results included in [11] on applications of approximation spaces to study properties of infomorphisms. Operations, called constrained sums, seem to be very important in searching for patterns in data mining [3] (e.g., in spatio-temporal reasoning) or in more general sense in generating relevant granules for approximate reasoning using calculi on information granules [7], [11].

The paper is organized as follows. In Section 2 we present basic concepts. In Section 3 we introduce sums of information systems and approximation spaces. In Section 4 we discuss constrained sums of information systems and hierarchical information systems.

2 Approximation Spaces and Infomorphisms

In this section we recall basic notions for our considerations.

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2.1 Approximation Spaces

We recall a general definition of an approximation space. Several known approaches to concept approximations can be covered using such spaces, e.g., the tolerance based rough set model or the variable precision rough set model.

For every non-empty set U, let $P(U)$ denote the set of all subsets of U.

Definition 1. *[8],[12] A parameterized approximation space is a system* $AS_{\#,\$} = (U, I_{\#}, \nu_{\$})$, *where*

- U *is a non-empty set of objects,*
- $I_{\#} : U \to P(U)$ *is an uncertainty function,*
- $\nu_{\$} : P(U) \times P(U) \to [0,1]$ *is a rough inclusion function,*

and $\#, \$$ *are denoting vectors of parameters.*

The uncertainty function defines for every object x a set of similarly described objects.

A set $X \subseteq U$ is *definable in* $AS_{\#,\$}$ if and only if it is a union of some values of the uncertainty function.

The rough inclusion function defines the degree of inclusion between two subsets of U (see, e.g., [8], [12]):

$$\nu_{SRI}(X, Y) = \begin{cases} \frac{card(X \cap Y)}{card(X)} & \text{if } X \neq \emptyset \\ 1 & \text{if } X = \emptyset. \end{cases}$$

This measure is widely used by data mining and rough set communities. However, Jan Łukasiewicz [4] was first who used this idea to estimate the probability of implications.

The lower and the upper approximations of subsets of U are defined as follows.

Definition 2. *For an approximation space* $AS_{\#,\$} = (U, I_{\#}, \nu_{\$})$ *and any subset* $X \subseteq U$ *the lower and the upper approximations are defined by*

$LOW\left(AS_{\#,\$}, X\right) = \{x \in U : \nu_{\$}(I_{\#}(x), X) = 1\}$,

$UPP\left(AS_{\#,\$}, X\right) = \{x \in U : \nu_{\$}(I_{\#}(x), X) > 0\}$, *respectively.*

2.2 Infomorphisms

In this section we recall the definition of infomorphism for two information systems [11]. We also present some new properties of infomorphisms. The infomorphisms for classifications are introduced and studied in [1].

We denote by $\Sigma(IS)$ the set of Boolean combinations of descriptors over IS and by $\|\alpha\|_{IS} \subseteq U$ is denoted the semantics of α in IS.

For all formulas $\alpha \in \Sigma(IS)$ and for all objects $x \in U$ we will denote $x \models_{IS} \alpha$ if and only if $x \in \|\alpha\|_{IS}$.

Definition 3. *[1, 11] If $IS_1 = (U_1, A_1)$ and $IS_2 = (U_2, A_2)$ are information systems then an infomorphism between IS_1 and IS_2 is a pair $(f^\wedge, f^\vee)$ of functions $f^\wedge : \Sigma(IS_1) \rightarrow \Sigma(IS_2)$, $f^\vee : U_2 \rightarrow U_1$, satisfying the following equivalence*

$$f^\vee(x) \models_{IS_1} \alpha \text{ if and only if } x \models_{IS_2} f^\wedge(\alpha), \tag{1}$$

for all objects $x \in U_2$ and for all formulas $\alpha \in \Sigma(IS_1)$.

The infomorphism will be denoted shortly by $(f^\wedge, f^\vee) : IS_1 \rightleftarrows IS_2$.

Proposition 1. *[11] For any infomorphism $(f^\wedge, f^\vee) : IS_1 \rightleftarrows IS_2$ we obtain the following equality*

$$(f^\vee)^{-1}(\|\alpha\|_{IS_1}) = \|f^\wedge(\alpha)\|_{IS_2} \text{ for any } \alpha \in \Sigma(IS_1). \tag{2}$$

Definition 4. *Let $(f^\wedge, f^\vee)$ be an infomorphism between IS_1 and IS_2. We define two binary relations $\sim_{f^\wedge} \subseteq \Sigma(IS_1) \times \Sigma(IS_1)$ and $\approx_{f^\vee} \subseteq U_2 \times U_2$ as follows*

1. *$\alpha \sim_{f^\wedge} \beta$ if and only if $f^\wedge(\alpha) = f^\wedge(\beta)$ for any $\alpha, \beta \in \Sigma(IS_1)$,*
2. *$x \approx_{f^\vee} y$ if and only if $f^\vee(x) = f^\vee(y)$ for any $x, y \in U_2$.*

We obtain the following proposition:

Proposition 2. *For any infomorphism $(f^\wedge, f^\vee) : IS_1 \rightleftarrows IS_2$ between IS_1 and IS_2 the following properties hold:*

1. *The relations $\sim_{f^\wedge}$ and $\approx_{f^\vee}$ are equivalence relations;*
2. *$\alpha \sim_{f^\wedge} \beta$ if and only if*

   $$\|\alpha\|_{IS_1} \cap f^\vee(U_2) = \|\beta\|_{IS_1} \cap f^\vee(U_2)$$

   for any $\alpha, \beta \in \Sigma(IS_1)$,
3. *$x \approx_{f^\vee} y$ if and only if*

   $$(x \in \|f^\wedge(\alpha)\|_{IS_2} \text{ if and only if } y \in \|f^\wedge(\alpha)\|_{IS_2}) \text{ for any } \alpha \in \Sigma(IS_1)$$

   where $x, y \in U_2$,
4. *either $[x]_{\approx_{f^\vee}} \subseteq \|f^\wedge(\alpha)\|_{IS_2}$ or $[x]_{\approx_{f^\vee}} \cap \|f^\wedge(\alpha)\|_{IS_2} = \emptyset$ for any $\alpha \in \Sigma(IS_1)$ and $x \in U_2$,*
5. *any formula $\alpha \in f^\wedge(\Sigma(IS_1))$ is crisp (definable) in $U_2/\approx_{f^\vee}$, i.e., $\|\alpha\|_{IS_2}$ is a union of some equivalence classes from $U_2/\approx_{f^\vee}$.*

Let us recall that formulas from $\Sigma(IS_2) - f^\wedge(\Sigma(IS_1))$ can be defined approximately in $U_2/\approx_{f^\vee}$ (see [11]).

Proposition 2 gives a characterization of infomorphisms.

Definition 5. *Let $(f^\wedge, f^\vee) : IS_1 \rightleftarrows IS_2$ be an infomorphism between IS_1 and IS_2. We define two information systems:*

$$IS_1' = (f^\vee(U_2), \Sigma(IS_1')) \text{ and } IS_2' = (U_2', \Sigma(IS_2'))$$

where

- *$\Sigma(IS_1')$ is a subset of $\Sigma(IS_1)$ consisting of exactly one element from each equivalence class from $\Sigma(IS_1)/\sim_{f^\wedge}$ and*
- *U_2' is a subset of U_2 consisting of exactly one element from each equivalence class from $U_2/\approx_{f^\vee}$ and $\Sigma(IS_2') = f^\wedge(\Sigma(IS_1))$.*

Proposition 3. *Let $(f^\wedge, f^\vee) : IS_1 \rightleftarrows IS_2$ be an infomorphism between IS_1 and IS_2. Then $(g^\wedge, g^\vee) : IS_1' \rightleftarrows IS_2'$ is an infomorphism where $(g^\wedge, g^\vee)$ is a pair of bijections defined by $g^\wedge(\alpha) = f^\wedge(\alpha)$ and $g^\vee(x) = f^\vee(x)$ for any $\alpha \in \Sigma(IS_1')$ and any $x \in U_2'$.*

In Proposition 3 we assume that $\|\alpha\|_{IS_2'} = \|\alpha\|_{IS_2} \cap U_2'$ for $\alpha \in \Sigma(IS_2')$. This proposition expresses that on domains accessible in communication (between two agents represented by information systems) established by a given infomorphism, the infomorphism is defined by selection functions on equivalence classes of formulas and objects, respectively. Such functions are bijections. From this fact it follows that, roughly speaking, infomorphisms of information systems can be realized by operations which we call constrained sums. The details are presented in the following sections.

However, observe that the communication established by infomorpshims do not assure the complete knowledge between communicating agents (information systems). In particular, formulas (concepts) from $\Sigma(IS_2) - f^\wedge(\Sigma(IS_1))$ are not in general definable in IS_1, only their approximations are known for IS_1 [11].

3 Sum of Information Systems and Approximation Spaces

3.1 Sum of Information Systems

In this section we introduce a sum of two information systems.

Definition 6. *Let $IS_1 = (U_1, A_1)$ and $IS_2 = (U_2, A_2)$ be information systems. These information systems can be combined into a single information system, denoted by $+(IS_1, IS_2)$, with the following properties:*

- *The objects of $+(IS_1, IS_2)$ consist of pairs (x_1, x_2) of objects from IS_1 and IS_2 i.e. $U = U_1 \times U_2$*
- *The attributes of $+(IS_1, IS_2)$ consist of the attributes of IS_1 and IS_2, except that if there are any attributes in common, then we make distinct copies, so as not to confuse them.*

Proposition 4. *There are infomorphisms $(f_k^\wedge, f_k^\vee) : IS_k \rightleftarrows +(IS_1, IS_2)$ for $k = 1, 2$ defined as follows:*

- *$f_k^\wedge(\alpha) = \alpha_{IS_k}$ (the IS_k-copy of α) for each $\alpha \in \Sigma(IS_k)$*
- *for each pair $(x_1, x_2) \in U$, $f_k^\vee((x_1, x_2)) = x_k$*

Given any information system IS_3 and infomorphisms $(f_{k,3}^\wedge, f_{k,3}^\vee) : IS_k \rightleftarrows IS_3$, there is a unique infomorphism $(f_{1+2,3}^\wedge, f_{1+2,3}^\vee) : +(IS_1, IS_2) \rightleftarrows IS_3$ such that in Figure 1 one can go either way around the triangles and get the same result.

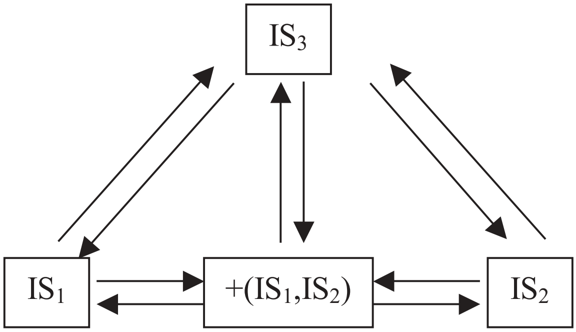

Fig. 1. Sum of Information Systems IS_1 and IS_2

Example 1. Let us consider a diagnostic agent testing failures of the space robotic arm. Such an agent should observe the arm and detect a failure if, e.g., some of its parts are in abnormal relative position. Assume, in our simple example that projections of some parts on a plane are observed and a failure is detected if projection of some parts that are triangles or rectangles are in some relation, e.g., the triangle is not included sufficiently inside of the rectangle. Hence, any considered object consists of parts: a triangle and a rectangle. Objects are perceived by some attributes expressing properties of parts and a relation (constraint) between them.

First, we construct an information system, called the sum of given information systems. Such system represents objects composed from parts without any constraint. It means that we consider as the universe of objects the Cartesian product of the universes of parts (Tables 1-3).

Let us consider three information systems $IS_{rectangle} = (U_{rectangle}, A_{rectangle})$, $IS_{triangle} = (U_{triangle}, A_{triangle})$, and $+(IS_{rectangle}, IS_{triangle}) = (U_{rectangle} \times U_{triangle}, \{(a,1),(b,1),(c,2)\})$ presented in Tables 1-3, respectively. Let $U_{rectangle}$ be a set of rectangles and $A_{rectangle} = \{a, b\}$, $V_a = [0, 300]$ and $V_b = \{yes, no\}$, where the value of a means a length in millimeters of horizontal side of rectangle and for any object $x \in U_{rectangle}$ $b(x) = yes$ if and only if x is a square.

Let $U_{triangle}$ be a set of triangles and $A_{triangle} = \{c\}$ and $V_c = \{t_1, t_2\}$, where $c(x) = t_1$ if and only if x is an acute–angled triangle and $c(x) = t_2$ if and only if x is a right–angled triangle.

We assume all values of attributes are made on a given projection plane. The results of measurements are represented in information systems. Tables 1-2 include only illustrative examples of the results of such measurements.

We assume that $(a,1)((x_i, y_j)) = a(x_i)$, $(b,1)((x_i, y_j)) = b(x_i)$ and $(c,2)((x_i, y_j)) = c(y_j)$, where $i = 1, \ldots, 6$ and $j = 1, 2$.

3.2 Sum of Approximation Spaces

In this section we present a simple construction of approximation space for the sum of given approximation spaces.

Table 1. Information System $IS_{rectangle}$ with Uncertainty Functions

$U_{rectangle}$	a	b	$I_a(\cdot)$	$I_b(\cdot)$	$I_{A_1}(\cdot)$
x_1	165	*yes*	$\{x_1, x_3, x_5, x_6\}$	$\{x_1, x_3\}$	$\{x_1, x_3\}$
x_2	175	*no*	$\{x_2, x_4, x_6\}$	$\{x_2, x_4, x_5, x_6\}$	$\{x_2, x_4, x_6\}$
x_3	160	*yes*	$\{x_1, x_3, x_5\}$	$\{x_1, x_3\}$	$\{x_1, x_3\}$
x_4	180	*no*	$\{x_2, x_4\}$	$\{x_2, x_4, x_5, x_6\}$	$\{x_2, x_4\}$
x_5	160	*no*	$\{x_1, x_3, x_5\}$	$\{x_2, x_4, x_5, x_6\}$	$\{x_5\}$
x_6	170	*no*	$\{x_1, x_2, x_6\}$	$\{x_2, x_4, x_5, x_6\}$	$\{x_2, x_6\}$

Table 2. Information System $IS_{triangle}$ with Uncertainty Function I_{A_2}

$U_{triangle}$	c	$I_{A_2}(\cdot)$
y_1	t_1	$\{y_1, y_3\}$
y_2	t_2	$\{y_2\}$
y_3	t_1	$\{y_1, y_3\}$

Let $AS_{\#_k} = (U_k, I_{\#_k}, \nu_{SRI})$ be an approximation space for information system IS_k, where $k = 1, 2$. We define an approximation space $+(AS_{\#_1}, AS_{\#_2})$ for information system $+(IS_1, IS_2)$ as follows:

1. the universe is equal to $U_1 \times U_2$,
2. $I_{\#_1,\#_2}((x_1, x_2)) = I_{\#_1}(x_1) \times I_{\#_2}(x_2)$,
3. the inclusion relation ν_{SRI} in $+(AS_{\#_1}, AS_{\#_2})$ is the standard inclusion function.

Proposition 5. *Let $X \subseteq U_1$ and $Y \subseteq U_2$. We have the following properties of approximations:*

$$LOW(+(AS_{\#_1}, AS_{\#_2}), X \times Y) = LOW(AS_{\#_1}, X) \times LOW(AS_{\#_2}, Y), \quad (3)$$
$$UPP(+(AS_{\#_1}, AS_{\#_2}), X \times Y) = UPP(AS_{\#_1}, X) \times UPP(AS_{\#_2}, Y). \quad (4)$$

Proof. We have $I_{\#_1,\#_2}((x_1, x_2)) \subseteq X \times Y$ iff $I_{\#_1}(x_1) \subseteq X$ and $I_{\#_2}(x_2) \subseteq Y$. Moreover, $I_{\#_1,\#_2}((x_1, x_2)) \cap (X \times Y) \neq \emptyset$ iff $I_{\#_1}(x_1) \cap X \neq \emptyset$ and $I_{\#_2}(x_2) \cap Y \neq \emptyset$.

Example 2. For information system $IS_{rectangle}$ we define an approximation space $AS_{A_1} = (U_{rectangle}, I_{A_1}, \nu_{SRI})$ such that $y \in I_a^5(x)$ if and only if $|a(x) - a(y)| \leq 5$. This means that rectangles x and y are similar with respect to the length of horizontal sides if and only if the difference of lengths is not greater than 5 millimeters. Let $y \in I_b(x)$ if and only if $b(x) = b(y)$ and $y \in I_{A_1}(x)$ if and only if $\forall_{c \in A_1} y \in I_c(x)$. Thus, we obtain uncertainty functions represented in the last three columns of Table 1. For information system $IS_{triangle}$ we define an approximation space as follows: $y \in I_{A_2}(x)$ if and only if $c(x) = c(y)$ (see the last column of Table 2). For $+(IS_{rectangle}, IS_{triangle})$ we obtain $I_{A_1,A_2}((x, y)) = I_{A_1}(x) \times I_{A_2}(y)$ (see the last column of Table 3).

Table 3. An Information System $+(IS_{rectangle}, IS_{triangle})$ with Uncertainty Function I_{A_1,A_2}

$U_{rectangle} \times U_{triangle}$	$(a,1)$	$(b,1)$	$(c,2)$	$I_{A_1,A_2}((\cdot,\cdot))$
(x_1, y_1)	165	*yes*	t_1	$\{x_1, x_3\} \times \{y_1, y_3\}$
(x_1, y_2)	165	*yes*	t_2	$\{x_1, x_3\} \times \{y_2\}$
(x_1, y_3)	165	*yes*	t_1	$\{x_1, x_3\} \times \{y_1, y_3\}$
(x_2, y_1)	175	*no*	t_1	$\{x_2, x_4, x_6\} \times \{y_1, y_3\}$
(x_2, y_2)	175	*no*	t_2	$\{x_2, x_4, x_6\} \times \{y_2\}$
(x_2, y_3)	175	*no*	t_1	$\{x_2, x_4, x_6\} \times \{y_1, y_3\}$
(x_3, y_1)	160	*yes*	t_1	$\{x_1, x_3\} \times \{y_1, y_3\}$
(x_3, y_2)	160	*yes*	t_2	$\{x_1, x_3\} \times \{y_2\}$
(x_3, y_3)	160	*yes*	t_1	$\{x_1, x_3\} \times \{y_1, y_3\}$
(x_4, y_1)	180	*no*	t_1	$\{x_2, x_4\} \times \{y_1, y_3\}$
(x_4, y_2)	180	*no*	t_2	$\{x_2, x_4\} \times \{y_2\}$
(x_4, y_3)	180	*no*	t_1	$\{x_2, x_4\} \times \{y_1, y_3\}$
(x_5, y_1)	160	*no*	t_1	$\{x_5\} \times \{y_1, y_3\}$
(x_5, y_2)	160	*no*	t_2	$\{x_5\} \times \{y_2\}$
(x_5, y_3)	160	*no*	t_1	$\{x_5\} \times \{y_1, y_3\}$
(x_6, y_1)	170	*no*	t_1	$\{x_2, x_6\} \times \{y_1, y_3\}$
(x_6, y_2)	170	*no*	t_2	$\{x_2, x_6\} \times \{y_2\}$
(x_6, y_3)	170	*no*	t_1	$\{x_2, x_6\} \times \{y_1, y_3\}$

4 Constrained Sums

In this section we consider operations on information systems that can be used in searching for hierarchical patterns. The operations are parameterized by constraints. Hence, in searching for relevant patterns one can search for relevant constraints and elementary information systems used to construct hierarchical patterns represented by constructed information systems.

4.1 Constrained Sums of Information Systems

In this section we consider a new operation on information systems often used in searching, e.g., for relevant patterns. This operation is more general than theta join operation used in databases [2]. We start from the definition in which the constraints are given explicitly.

Definition 7. *Let $IS_i = (U_i, A_i)$ for $i = 1, \ldots, k$ be information systems and let R be a k-ary constraint relation in $U_1 \times \ldots \times U_k$, i.e., $R \subseteq U_1 \times \ldots \times U_k$. These information systems can be combined into a single information system relatively to R, denoted by $+_R(IS_1, \ldots, IS_k)$, with the following properties:*

- *The objects of $+_R(IS_1, \ldots, IS_k)$ consist of k-tuples $(x_1, \ldots, x_k)$ of objects from R, i.e., all objects from $U_1 \times \ldots \times U_k$ satisfying the constraint R.*
- *The attributes of $+_R(IS_1, \ldots, IS_k)$ consist of the attributes of $IS_1, \ldots, IS_k$, except that if there are any attributes in common, then we make distinct copies, so as not to confuse them.*

Usually the constraints are defined by conditions expressed by Boolean combination of descriptors of attributes (see Section 2.2). It means that the constraints are built from expressions $a\ in\ V$, where a is an attribute and $V \subseteq V_a$, using propositional connectives $\wedge$, $\vee$, $\neg$. Observe, that in the constraint definition we use not only attributes of parts (i.e., from information systems $IS_1, \ldots, IS_k$) but also some other attributes specifying relation between parts. In our example (see Table 4), the constraint R_1 is defined as follows: *the triangle is sufficiently included in the rectangle.* Any row of this table represents an object (x_i, y_j) composed of the triangle y_j included sufficiently into the rectangle x_i.

Let us also note that constraints are defined using primitive (measurable) attributes different than those from information systems describing parts. This makes the constrained sum different from the theta join [2]. On the other hand one can consider that the constraints are defined in two steps. In the first step we extend the attributes for parts and in the second step we define the constraints using some relations on these new attributes.

Let us observe that the information system $+_R(IS_1, \ldots, IS_k)$ can be also described using an extension of the sum $+(IS_1, \ldots, IS_k)$ by adding a new binary attribute that is the characteristic function of the relation R and by taking a subsystem of the received system consisting of all objects having value one for this new attribute.

The constraints used to define the sum (with constraints) can be often specified by information systems. The objects of such systems are tuples consisting of objects of information systems that are arguments of the sum. The attributes describe relations between elements of tuples. One of the attribute is a characteristic function of the constraint relation (restricted to the universe of the information system). In this way we obtain a decision system with the decision attribute defined by the characteristic function of the constraint and conditional attributes are the remaining attributes of this system. From such decision table one can induce classifier for the constraint relation. Next, such classifier can be used to select tuples in the construction of constrained sum.

Example 3. Let us consider three information systems $IS_{rectangle} = (U_{rectangle}, A_{rectangle})$, $IS_{triangle} = (U_{triangle}, A_{triangle})$, $+_{R_1}(IS_{rectangle}, IS_{triangle})$, presented in Table 1, Table 2 and Table 4, respectively. We assume that $R_1 = \{(x_i, y_j) \in U_{rectangle} \times U_{triangle} : i = 1, \ldots, 6 \quad j = 1, 2\}$. We also assume that $a'((x_i, y_j)) = a(x_i)$, $b'((x_i, y_j)) = b(x_i)$ and $c'((x_i, y_j)) = c(y_j)$, where $i = 1, \ldots, 6$ and $j = 1, 2$.

The above examples are illustrating an idea of specifying constraints by examples. Table 4 can be used to construct a decision table partially specifying characteristic functions of the constraint. Such a decision table should be extended by adding relevant attributes related to the object parts making it possible to induce the high quality classifiers for the constraint relation. The classifier can be next used to filter composed pairs of objects that satisfy the constraint. This is important construction because the constraint specification usually cannot be defined directly in terms of measurable attributes. It can be specified, e.g., in

Table 4. Information System $+_{R_1}(IS_{rectangle}, IS_{triangle})$

$(U_{rectangle} \times U_{triangle}) \cap R_1$	a'	b'	c'
(x_1, y_1)	165	*yes*	t_1
(x_1, y_2)	165	*yes*	t_2
(x_2, y_1)	175	*no*	t_1
(x_2, y_2)	175	*no*	t_2
(x_3, y_1)	160	*yes*	t_1
(x_3, y_2)	160	*yes*	t_2
(x_4, y_1)	180	*no*	t_1
(x_4, y_2)	180	*no*	t_2
(x_5, y_1)	160	*no*	t_1
(x_5, y_2)	160	*no*	t_2
(x_6, y_1)	170	*no*	t_1
(x_6, y_2)	170	*no*	t_2

natural language. This is the reason that the process of inducing of the relevant classifiers for constraints can require hierarchical classifier construction [6].

The constructed constrained sum of information systems can consists of some incorrect objects. This is due to not proper filtering of objects by the classifier for constraints induced from data (with accuracy usually less than 100%). One should take this issue into account in constructing nets of information systems.

4.2 Constrained Sum of Approximation Spaces

Let $AS_{\#_i} = (U_i, I_{\#_i}, \nu_{SRI})$ be an approximation space for information system IS_i, where $i = 1, \ldots, k$ and let $R \subseteq U_1 \times \ldots \times U_k$ be a constraint relation. We define an approximation space $+_R(AS_{\#_1}, \ldots, AS_{\#_k})$ for $+_R(IS_1, \ldots, IS_k)$ as follows:

1. the universe is equal to R,
2. $I_{\#_1,\ldots,\#_k}((x_1, \ldots, x_k)) = (I_{\#_1}(x_1) \times \ldots \times I_{\#_k}(x_k)) \cap R$,
3. the inclusion relation ν_{SRI} in $+_R(AS_{\#_1}, \ldots, AS_{\#_k})$ is the standard inclusion function.

Proposition 6. *Let $X_i \subseteq U_i$ for $i = 1, \ldots, k$. We obtain the following properties of approximations:*

$$LOW(+_R(AS_{\#_1}, \ldots, AS_{\#_k}), X_1 \times \ldots \times X_k) = R \cap (LOW(AS_{\#_1}, X_1) \times \ldots \times LOW(AS_{\#_k}, X_k)) \quad (5)$$

$$UPP(+_R(AS_{\#_1}, \ldots, AS_{\#_k}), X_1 \times \ldots \times X_k) = R \cap (UPP(AS_{\#_1}, X_1) \times \ldots \times UPP(AS_{\#_k}, X_k)). \quad (6)$$

Conclusions

In many cases the constraint relations are soft relations. Hence, they can be defined as fuzzy or rough relations. Properties of sums of information systems constructed relatively to soft constraints will be the subject of our further study. Moreover, hierarchical construction of patterns and classifier based on the introduced foundations is another interesting topic to study toward approximate reasoning in distributed or multiagent systems.

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