Chapter

Conflict Analysis

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1 Introduction

Computer support for different human activities has grown up in the latest years. Actually the researchers in Artificial Intelligence benefit from this fact in many fields not considered some years ago. Conflict analysis is one of the fields whose importance is increasing nowadays as distributed systems of computers are starting to play a significant role in the society. The computer aided conflict analysis must be applied when intelligent machines (agents) interact. However this is only one from many different areas where a conflict can arise like business, government, political or military operations, labour–management negotiations etc.etc.

In this Chapter, we first examine nature of conflicts as we are formally defining the conflict situation model. Then we investigate a consensus problem and we discuss the methods of solving it. Finally, other problems related to resolving conflicts are analysed.

The introduced model enhances the one proposed and investigated by Pawlak in [24, 26, 27, 28]. The new model is based on information stored in the information tables and on constrains among agents' attributes. Any data table may form a Pawlak information system and constraints are usually given in the form of boolean formulas. Boolean reasoning has been found as the best tool for conflict analysis within this model.

2 Pawlak Model

The simple model introduced by Pawlak [24] forms the basis for the model presented in this paper. In the Pawlak model, attitudes of agents to specific issues are depicted in the form of a table in which agents are represented by rows and issues by columns. The value assigned to each agent and to each attribute (issue) is in the set {-1, 0, 1}, where -1, 0, 1 mean, respectively, that an agent is *against*, *neutral* and *favourable* toward the issue.

Formally, any table described above can be represented as an *information system* defined as follows:

An information system is a pair $S = (U, A)$, where

U - is a nonempty, finite set called the universe; elements of U are called objects (here agents),

A - is a nonempty, finite set of attributes (issues).

Every attribute $a \in A$ is a map, $a : U \rightarrow V_a$, where the set V_a is the value set of a; elements of V_a are referred to as *opinions*, i.e. $a(x)$ is the opinion of an agent x about an issue a. The domain of each attribute (for the conflict analysis model) is restricted to three values only, i.e. $\mathrm{V}_a = \{-1, 0, 1\}$, which means against, neutral and favourable, respectively.

Example 1. Let us first analyse a conflict between an employer and an employee. The example is taken from the author's observation and is used to present the defined notions rather then resolve a real conflict. Job attributes considered for the worker are compensation and work conditions. On the other hand, employers are interested in factory profit, good investment level and, maybe, worker's satisfaction. We can think about these attributes quite generally, for example, compensation can consist of the worker's salary and all his income but it also can include the repeated profit division like the social fund. Similarly worker's conditions include a modern and safe work place and in addition a nice team and development possibilities. One can easily find that these aspects contradict each other in this example. We analyse this problem more deeply in the sequel.

Let us choose the issues for the Pawlak model (agents are voting on):

a - increasing the employees' incomes,

b - improving the work conditions,

c - increase the factory profit by reducing the costs of work,

d - increase the level of investment.

Then, the information table: Table 1, where ag_1 is the employee and ag_2 is the employer, can describe the conflict situation.

	a		c	d
ag_1	1	1	-1	0
ag_2	-1	0	1	1

Table 1. The conflict situation in the Pawlak model

The tension of the conflict [28] in the described situation can be calculated as equal to 1.

Analysis of conflicts described by the Pawlak model is restricted to outermost conclusions like finding the most conflicting attributes or the coalitions of agents if more than two take part in the conflict [8].

Because in the Pawlak model the reason for the conflict cannot be determined, there is no way to specify the situation for avoiding the conflict. Moreover, we cannot be sure that the issues the agents vote represent the issues each agent takes care of. In the next section, we will define a model allowing for answering the following basic questions.

- What are the conflict reasons?
- How the consensus can be found?
- Is it possible to satisfy all the agents?

3 New Model

In the Pawlak model, conflicts are presented at the outermost level. Some issues are chosen, and the agents are asked to specify their views: are they favourable, neutral or against. In the real world, views on the issues to vote are consequences of the decision taken, based on the local issues, the current state and some background knowledge. Therefore, the Pawlak model is enhanced here by adding to this model some local aspects of conflicts. The introduced model also gives a possibility to check if the issues to vote are chosen correctly, i.e., if the local issues determine the decisions.

3.1 Local states

The information about the local states U_{ag} of an agent ag can be represented in the form of an information table, creating the agent ag's information system $I_{ag} = (U_{ag}, A_{ag})$ where $a : U_{ag} \rightarrow V_a$ for any $a \in A_{ag}$ and V_a is the value set of attribute a. We assume:

$$V(ag) = \bigcup_{a \in A(ag)} V_a)$$

Any local state $s \in U_{ag}$ is explicitly described by its *information vector* $Inf_{A_{ag}}(s)$, where $Inf_{A_{ag}}(s)= \{(a, a(s)) : a \in A_{ag}\}$. The set $\{Inf_{A_{ag}}(s) : s \in U_{ag}\}$ is denoted by $INF_{A_{ag}}$ and it is called the *information vector set of* ag. We assume that sets $\{A_{ag}\}$ are pairwise disjoint, i.e., $A_{ag}) \cap A_{ag'} = \emptyset$ for $ag \neq ag'$. This condition emphasises that any agent is describing the situation in its own way. The manner of understanding the *same world* by each agent can be completely different. Relationships among attributes of different agents will be defined by constraints as shown in Section 3.3.

Example 2 illustrates local states for the labour–management conflict.

Subjective evaluation of local states (similarity of states) Every agent has favourable (target) states in the set of local states, i.e., those states the agent wants to reach. In the information table of ag the states the agent ag cannot accept can also appear; being in such a state could mean a disaster for the agent. Actually, the agent evaluates each state. The *subjective evaluation* corresponds to an order (or partial order) of the states of the agent information table. We assume that the function e_{ag} called the *target function*, assigns an evaluation score to each state. An exemplary target function used in our examples is defined to be a function $e_{ag} : U_{ag} \rightarrow R[0,1]$. The states with score 1 are mostly preferred by the agent as target states, while the states with score 0 are not acceptable.

The current information vector of ag is a vector describing the current local state of the agent ag.

The state evaluation can also help us to find the state similarity (see e.g. [30] for references on similarity in rough set investigations). For any $\varepsilon > 0$ and $s \in U_{ag}$, we define *ε-neighbourhood* of s by:

$$\tau_{ag,\varepsilon}(s) = \{s\prime \in U_{ag} : |e_{ag}(s) - e_{ag}(s')| \leq \varepsilon\}$$

The family $\{\tau_{ag,\varepsilon}(s)\}_{s \in U_{ag}}$ defines a tolerance relation $\tau_{ag,\varepsilon}$ in $U_{ag} \times U_{ag}$ by $s\tau_{ag,\varepsilon}s\prime$ iff $s\prime \in \tau_{ag,\varepsilon}(s)$.

Example 2. Let us consider the situation described in Example 1. Ag consists of two agents: ag_1– the employee and ag_2–the employer.

Table 2 shows agent's ag_1 states, i.e., views on local issues (attributes) a, b and the state subjective evaluation.

a–compensation,
b–work conditions

local states	a		evaluation e_{ag1}
s1	2	2	1
s2 (current)	2	1	$\frac{2}{3}$
s3	1	2	$\frac{1}{3}$
s4	1	1	0
s5	2	0	0

Table 2. Local states of agent ag_1 with su jective evaluation

The agent ag_2 describes its view on local issues k, l, m where
k–factory profit,
l–level of investment,
m–worker's satisfaction.

local states	k	l	m	evaluation e_{ag2}
s1	2	2	2	1
s2	1	2	2	$\frac{2}{3}$
s3	1	1	2	$\frac{1}{3}$
s4	1	1	1	$\frac{1}{3}$
s5 (current)	2	0	1	0

Table 3. Local states of agent ag_2 with su jective evaluation

For simplicity, let us assume that attributes' domains for both agents are the same, and values belong to the set $V = \{0, 1, 2\}$. One can interpret the values from set V as *small*, *medium* and *high* levels, respectively. For example, the state s_1 of the agent ag_1 expresses a high level of compensation and high level of work conditions. Similarly, the state s_3 of the agent ag_2 means medium profit level with medium level of investment, while at the same time worker satisfaction is high.

Distance function A tolerance relation τ describes similarity of states according to the subjective evaluation. However, it is necessary to describe the state similarity according to differences between values of attributes. Similarity of states from U_{ag} can be often defined as follows.

We assume that for any $a \in A_{ag}$ there is a distance function:

$$d_a : U_{ag} \times U_{ag} \rightarrow \ R+$$

For example, $d_a(s, s\prime) = |a(s) - a(s\prime)|$ if $V_{ag}(a) \subseteq R$.

Next we define the distance function

$$d : U_{ag} \times U_{ag} \rightarrow \ R+ \text{ by } d(s, s\prime) = F(d_{a1}(s, s\prime), ..., d_{am}(s, s\prime))$$

where $A_{ag} = \{a1, ..., am\}$ and $F : R_+^m \rightarrow \ R+$ is a function like e.g.

$$F(r1, ..., rm) = \sqrt{r_1^2 + ... + r_m^2}$$

The function F depends on the problem and should be chosen reflecting the problem specificity.

Crucial for the negotiation process results and for ability to solve any conflict is agents' willingness to change the current state (possibly giving up some resources). This disposition is the basis to define *closeness* of states agents are ready to accept. Closeness is defined by a distance function in the following manner: two states s and s' are close iff $d(s, s\prime) < \varepsilon_{ag}$, where ε_{ag} is a given threshold for ag. Consequently, a closeness neighbourhood of the state s with a diameter ε_{ag} is defined by $\{s\prime : d(s, s\prime) < \varepsilon_{ag})\}$.

Example 3. Let $\varepsilon(\mathrm{ag}_2)=\frac{2}{3}$. The example of closeness neighbourhood of the local state s_2 with the diameter $\frac{2}{3}$ is presented in Table 4.

We assume $F(v1, v2, v3) = \frac{1}{3}(v1 + v2 + v3)$ and

$$d(s2, s2_1) = F(d_k(s2, s2_1), d_l(s2, s2_1), d_m(s2, s2_1)).$$

Hance $d(s2, s2_1) = \frac{1}{3}(1 + 0 + 0) = \frac{1}{3}$.

local states	k	l	m
s2	1	2	2
$s2_1$	2	2	2
$s2_2$	1	1	2
$s2_3$	1	2	1

Table 4. The closeness of state 2 within the threshold $\frac{2}{3}$

Local set of goals (targets) The target function introduces a partial order in the set of local states so that one can find the maximal element(s) (with the highest evaluation) and the minimal one(s). Maximal elements can be interpreted as those, which are targets of the agent, i.e., the agent wants to reach them e.g. in a negotiation process. The agent ag's *set of goals (targets)* denoted by $T(ag)$ is defined as the set of target states of ag, which means

$$T(ag) = \{s \in U_{ag} : e_{ag}(s) > \mu_{ag}\}$$

and μ_{ag} is the boundary level, chosen by the agent ag - it is subjective which evaluation level is acceptable by the agent.

Example 4. In the considered situation, the minimal acceptable level of evaluation by the both agents will be, e.g., a score greater than $\frac{1}{3}$. Accordingly sets of goals of agents ag_1 and ag_2 are as follows: $T(ag_1)=\{s1, s2\}$ and $T(ag_2)=\{s1, s2\}$.

The set of goals can also be presented in the propositional form. The information table with scores is going to be converted to the decision table in which the decision 1 means that the state belongs to the set of goals, while 0 that it does not. Then the rules for decision 1 are found (for the method of rule generation see e.g. [29]).

The decision table of an agent ag_1 with the threshold $\frac{1}{3}$ is constructed and presented in Table 5.

states	a		decision d
s1	2	2	1
s2 (current)	2	1	1
s3	1	2	0
s4	1	1	0
s5	2	0	0

Table 5. The local set of goals - the decision ta le

Rule for $_1$: $(a_2 \wedge b_2) \vee (a_2 \wedge b_1) \rightarrow \; d_1$
Rule for d_0: $(a_1 \wedge b_2) \vee (a_1 \wedge b_1) \vee b_0 \rightarrow \; d_0$

Remark. We consider rules minimal with respect to the number of descriptors on the left-hand side (see e.g. [20, 29] for references to decision rules generation), i.e., they can be used to specify the new decisions for the states not yet included in the table.

Remark. In the rest of the paper, the parentheses are omitted in boolean expressions, according to the rule that the conjunction operator binds more strongly than that of disjunction. Thus, the expression of the form $a \wedge b \vee g \wedge d$ is understood as $(a \wedge b) \vee (g \wedge d)$. Furthermore, the conjunction sign $\wedge$ will be omitted in long formulas. Boolean variables like a_2 are understood as $a = 2$.

3.2 Situation

Let us consider a set Ag consisting of n agents ag_1,...,ag_n. A *situation* of Ag is any element of the Cartesian product $S(Ag) = \prod_{i=1}^{n} INF^*(ag_i)$, where $INF*(ag_i)$ is the set of all possible information vectors of the agent ag_i, defined by

$$INF^*(ag) = \{f : A(ag) \rightarrow \bigcup_{a \in A(ag)} V_a(ag) : f(a) \in V_a(ag) \ for \ a \in A(ag)\}$$

The situation $\bar{s}(Ag) \in S(Ag)$ corresponding to the global state

$$\bar{s} = (s_1, ..., s_n) \in U_{ag_1} \times ... \times U_{ag_n}$$

is defined by $(Inf_{A_{ag_1}}(s_1), ..., Inf_{A_{ag_n}}(s_n))$.

Example 5. An example of a current situation is the one presented in Table 6.

	a		k	l	m
current	2	1	2	0	1

Table 6. Current situation

3.3 Constraints

Constraints are described by some dependencies among local states of agents. Without any dependencies, any agent could take the next state freely. If there is no influence of a given agent on states of other agents – there is no conflict at all. Dependencies among local states of agents come, e.g., from the bound on the number of resources (any kind of a resource may be considered, e.g. water on Golan Hills see [24] or an international position [21], everything that is essential for agents). Constraining relations are introduced to express which local states of agents can coexist in the (global) situation. More precisely, *constraints* are used to define a subset $S(Ag)$ of global situations.

Constraints restrict the set of possible situations to admissible situations satisfying constraints. We will consider only admissible situations (shortly, situations) in the rest of the paper.

Example 6. The following dependencies restrict the set of situations and are constraints in our example. Attribute names here stand for the variables corresponding to attribute values. Constants here have been taken experimentally to express relationships and to allow comparison of any two variables.

1. $a > 0$ (compensation must be medium at least)
2. $1.5 + k > a + l$ (division of profit – a very simple case, i.e., the company uses its current profit for all expenses)
3. $2.5 + m > a + b$ (workers' satisfaction comes from a good level of salary and work conditions)

Constraints above can be converted to propositional formulas ($f_{\varphi 1}$, $f_{\varphi 2}$ and $f_{\varphi 3}$), respectively. The conjunction of formulas $f_{\varphi}= f_{\varphi 1} \wedge f_{\varphi 2} \wedge f_{\varphi 3}$ defines all admissible situations in our example. Let us see how formulas $f_{\varphi 1}$, $f_{\varphi 2}$ and $f_{\varphi 3}$ are created.

The equation $a > 0$ yields the formula $f_{\varphi 1}=a_1 \vee a_2$. The next formula (from the equation $1.5 + k > a + l$) is much more complex:

$f_{\varphi 2} = k_0a_0l_0 \vee k_0a_0l_1 \vee k_0a_1l_0 \vee k_1a_0l_0 \vee k_1a_0l_1 \vee k_1a_0l_2 \vee k_1a_1l_0 \vee k_1a_1l_1 \vee k_1a_2l_0 \vee k_2a_0l_0 \vee k_2a_0l_1 \vee k_2a_0l_2 \vee k_2a_1l_0 \vee k_2a_1l_1 \vee k_2a_1l_2 \vee k_2a_2l_0 \vee k_2a_2l_1$

The formula $f_{\varphi 3}$ is created in a similar way:

$f_{\varphi 3}=m_0a_0b_0 \vee m_0a_0b_1 \vee m_0a_0b_2 \vee m_0a_1b_0 \vee m_0a_2b_0 \vee m_0a_1b_1 \vee m_1a_0b_0 \vee m_1a_0b_1 \vee m_1a_0b_2 \vee m_1a_1b_0 \vee m_1a_2b_0 \vee m_1a_1b_1 \vee m_1a_1b_2 \vee m_1a_2b_1 \vee m_2a_0b_0 \vee m_2a_0b_1 \vee m_2a_0b_2 \vee m_2a_1b_0 \vee m_2a_2b_0 \vee m_2a_1b_1 \vee m_2a_1b_2 \vee m_2a_2b_1 \vee m_2a_{22}$

As already mentioned, constraints describe the situations that are admissible i.e. all local states can coexist in the admissible situation. For example, the situation $a = 2$, $b = 2$, $k = 2$, $l = 2$, $m = 2$ is not admissible because of constraints 2 and 3. The set of all admissible situations is described by the prime implicants of the boolean formula $f_{\varphi} = f_{\varphi 1} \wedge f_{\varphi 2} \wedge f_{\varphi 3}$.

3.4 Objective evaluation of situations

Agents could possibly not care about the global good. However, the real consensus (a non-conflicting situation) can be found only when the global good is taken into account by all participants [21]. Thus, the objective evaluation of situations is introduced to score on situations. More precisely the *quality function of the situations* is the function $q : S(Ag) \rightarrow R[0, 1]$ which assigns the evaluation score to each situation, where $S(Ag)$ is the set of all admissible situations. The score function specification can be as follows. An expert could give the score of some situations. Next, rules can be generated for different degrees of the score function value.

The set of situations satisfying a given level of quality t is defined by:

$$Score_{Ag}(t) = \{\overline{s} \in \prod_{ag \in Ag} U_{ag} : q(\overline{s}) \geq t\}$$

Example 7. Values of the function q and some admissible situations (these scored by an expert) of our example are presented in Table 7.

Situations	a		k	l	m	q(S)	decision
S1	1	1	2	2	1	$\frac{2}{3}$	1
S2	1	0	0	0	2	0	0
S3	1	0	1	0	2	0	0
S4	1	0	1	1	2	$\frac{1}{3}$	0
S5	1	0	2	0	2	0	0
S6	1	0	2	1	2	$\frac{1}{3}$	0
S7	1	0	2	2	2	$\frac{1}{3}$	0
S8	1	1	0	0	2	0	0
S9	1	1	1	0	2	0	0
S10	1	1	1	1	2	$\frac{1}{3}$	0
S11	1	1	2	0	2	$\frac{1}{3}$	0
S12	1	1	2	1	2	$\frac{1}{3}$	0
S13	1	1	2	2	2	$\frac{2}{3}$	1
S14	1	2	0	0	2	0	0
S15	1	2	1	0	2	0	0
S16	1	2	1	1	2	$\frac{1}{3}$	0
S17	1	2	2	0	2	0	0
S18	1	2	2	1	2	$\frac{2}{3}$	1
S19	1	2	0	0	1	0	0
S20	1	2	1	0	1	0	0
S21	1	2	1	1	1	$\frac{1}{3}$	0
S22	1	2	2	1	1	$\frac{2}{3}$	1
S23	2	1	2	1	2	$\frac{2}{3}$	1
S24	2	2	2	1	2	$\frac{2}{3}$	1
S25	1	2	2	2	1	1	1
S26 (current)	2	1	2	0	1	0	0

Table 7. Admissi le situations with the quality score

Let us find minimal rules for admissible situations with the quality score not lower than $\frac{2}{3}$ – these rules are going to be used in calculations in the next section.

$b_1l_2 \vee b_2l_2 \vee a_1b_1m_1 \vee a_1k_2m_1 \vee l_2m_1 \vee b_2k_2l_1 \vee b_2k_2m_1 \vee k_{21}m_1 \vee a_2l_1 \vee a_2m_2 \vee a_2b_2 \rightarrow q(S) \geq \frac{2}{3}$

3.5 Global preference function vs. objective situation evaluation

Though the situation is objectively evaluated by the quality function the influence of local preferences (defined by subjective evaluation) onto the global situation evaluation has to be outlined. One solution is to consider local preferences while looking for the consensus (Problem 5.2). The other solution is to express the global situation evaluation based on local preferences. For this purpose the global preference function is introduced in this section, which passes the local states evaluation onto the level of the global situation. The consensus reached based on the global preference function denotes the agreement between agents (found without an expert's help). Such a consensus is usually more stable but might be objectively worse than the one proposed by an expert – the agents may not take care about the global good.

Global preference function The global preference function for the admissible situation S corresponding to the global state $\overline{s}$=$(s_1, ..., s_n)$ can be defined by:

$$p(\overline{s} = (s_1, s_2, ..., s_n)) = F(e_{ag_1}(s_1), e_{ag_2}(s_2), ..., e_{ag_n}(s_n))$$

where F is a suitable, chosen function e.g. $F(r_1, ..., r_m) = \sum_{i=1}^{m} r_i$.

Consequently the set of all acceptable global situations is defined by:

$$S_{accept_{Ag}}(t) = \{S : p(S) \geq t\}, \text{ where } t \text{ is a given threshold.}$$

Remark. We assume that agents express evaluations of local states in the same way (they use the same scale).

Remark. The function used in this Chapter is very simple, however the global evaluation can be described in any suitable form (also non–linear) like in the form of decision rules.

3.6 System with constraints

The multi–agent system, with local states for each agent defined and the global situations satisfying constraints, will be called *the system with constraints.* We denote our system with constraints by M_{Ag}.

4 Conflict definition

In Section 3.1-3.6 the system with constraints has been defined. In such systems, conflict can be defined on several different levels.

4.1 Local conflict

The agent ag is in the ε-*local conflict* in a state s iff s does not belong to the ε-neighbourhood of s', for any s' from the set of ag-targets where ε is a given threshold.

Local conflicts for an agent ag arise from the low level of subjective evaluation of the current state by ag. The value $Cl_{ag}(s)$, which can be treated as a degree of the local conflict for ag at $s \in U_{ag}$ is defined by

$$Cl_{ag}(s) = \{ \begin{matrix} f_{ag}(s) - \varepsilon, \ when \ f_{ag}(s) > \varepsilon \\ 0, \ otherwise \end{matrix}$$

where ε is a given threshold. The function f_{ag} evaluates the distance from the state s to the set of targets of ag, i.e. $f_{ag}(s) = \min\{|e_{ag}(s) - e_{ag}(s\prime)| : s\prime \in T(ag)\}$, where $e_{ag}(s)$ is the subjective evaluation by ag at the local state s.

Example 8. We choose the threshold ε in our example to be equal 0, i.e., we want to obtain states without any local conflict. For the state s_2 of the agent ag_1, $Cl_{ag1}(s2) = 0$ – the current state belongs to the set of targets. However, $Cl_{ag2}(s5)=\frac{2}{3}-0=\frac{2}{3}$ i.e. the agent ag_2 is in a local conflict at s_5, the current state s_5 is not satisfactory for agent ag_2.

4.2 Global conflict

Global conflict can be measured by applying the global preference function or based on an expert evaluation. The difference lies in the way of considering the global good (see Section 3.5 for explanation).

Global conflict (based on an expert evaluation) A situation S is called t–*objectively conflicting* for Ag where t is a given threshold iff S does not belong to the set $Score_{Ag}(t)$. When the current situation is conflicting for Ag then agents from Ag are in the *objective global conflict*. The difference between the situation score and the given threshold can be treated as a global conflict degree, i.e.,

$$Cg_{ag}(S) = \{ \begin{matrix} t - q(S), \ when \ t > q(S) \\ 0, \ otherwise \end{matrix}$$

where t is the given threshold and q is the quality function.

Example 9. In discused example, let us take t=$\frac{2}{3}$. The current situation S26 is t-conflicting for $\{ag_1, ag_2\}$ and the global conflict factor is equal to Cg(S26)=$\frac{2}{3}$-0=$\frac{2}{3}$.

Global conflict (based on agents preferences) Consequently, a situation S is called $t - conflicting$ for Ag where t is a given threshold iff S does not belong to the set $S_{accept_{Ag}}(t)$. When the current situation is conflicting for Ag then agents from Ag are in the global conflict. The difference between the situation score and the given threshold can be treated as this kind of conflict degree, i.e.,

$$Cp_{ag}(S) = \{ \begin{matrix} t - p(S), \ when \ t > p(S) \\ 0, \ otherwise \end{matrix}$$

where t is the given threshold and p is the global preference function.

5 Problems

The introduced above conflict model gives us possibility, first to understand and, then, to analyse different kinds of conflicts. Particularly, the most fundamental problem is widely investigated, that is, the possibility to achieve the consensus. As in everyday's life, the consensus can be found on several levels and under some conditions.

5.1 Consensus

The consensus problem can be defined as follows.

`INPUT`

The system with constraints M_{Ag} defined in Section 3.
t - an acceptable threshold of the objective global conflict for Ag.

`OUTPUT`

The set of all situations with eliminated global conflict i.e., $Cg_{Ag}(S') = 0$, where S' is any new, reconstructed situation. That means that the quality score of the new, reconstructed situation cannot be lower then the given threshold t.

`ALGORITHM`

The algorithm must analyse all admissible situations and find these with the quality score not lower than the given threshold t.

Finding the solution consists in retrieving the formula which describes the set $Score_{Ag}(t)$ and verifying it against constraints (not all admissible situations have to be considered by an expert). To do this, the information table with scored situations is converted into a decision table. We are looking for a formula (rule) describing the decision that the situation is not conflicting. How to create such a formula has been shown in Example 3.6. Finally, the formula f_N describing the consensus problem is as follows:

$f_N = f_C \wedge f_\varphi$

where f_C is the formula which describes the set $Score_{Ag}(t)$ and f_φ describes constraints.

One can find that changing the global situation does not solve all the problems. The quality of local states of agents is not considered – the local conflict can be even stronger then before. In the sections that fallow, we are going to analyse this problem more deeply and we will try to eliminate local conflicts as well.

Example 10. The formula f_C for our conflict with a delimiter $t = \frac{2}{3}$ has been created in Example 7 and the formula f_φ in Example 6. Calculations give us the following formula f_N in the normal disjunctive form. Each prime implicant denotes the proposal for a non-conflicting situation.

$$f_N = a_1b_1k_2l_2m_0 \vee a_1b_1k_2l_2m_1 \vee a_1b_1k_2l_2m_2 \vee a_1b_2k_2l_2m_1 \vee a_1b_2k_2l_2m_2 \vee a_1b_0k_2l_2m_1 \vee a_2b_0k_2l_1m_0 \vee a_2b_0k_2l_1m_1 \vee a_2b_1k_2l_1m_1 \vee a_2b_0k_2l_1m_2 \vee a_2b_1k_2l_1m_2 \vee a_2b_2k_2l_1m_2 \vee a_2b_0k_1l_0m_2 \vee a_2b_1k_1l_0m_2 \vee a_2b_2k_1l_0m_2 \vee a_2b_0k_2l_0m_2 \vee a_2b_1k_2l_0m_2 \vee a_2b_2k_2l_0m_2 \vee a_1b_1k_0l_0m_1 \vee a_1b_1k_1l_0m_1 \vee a_1b_1k_1l_1m_1 \vee a_1b_1k_2l_0m_1 \vee a_1b_1k_2l_1m_1 \vee a_1b_0k_2l_0m_1 \vee a_1b_2k_2l_0m_1 \vee a_1b_0k_2l_1m_1 \vee a_1b_2k_2l_1m_1 \vee a_1b_2k_2l_1$$

Remark. It could be noticed that not every resulting non–conflicting situation has been scored (considered) by an expert. The method for rule generation applied here allows for searching for the solution within the equivalence classes of decisions – we cannot request the expert to specify all admissible situations. Furthermore, a useful information about agents' behavior can be achieved in this way from the historical data (like previous conflicts).

Remark. All calculations here have been done with the program module created by the author. Without computer aid this kind of analysis would be practically impossible. More about calculations can be found in Section 8.

5.2 Consensus on local preferences

In this section a conflict analysis is proposed where local information tables and the set of local goals are taken into consideration.

`INPUT`

The system with constraints M_{Ag} defined in Section 3.
t - an acceptable threshold of the objective global conflict for Ag.

`OUTPUT`

All situations with the objective evaluation reduced to degree at most t, and without local conflict for any agent.

The problem in this section consists in looking for a better compromise: additionally it is required that any new situation is constructed in the way that all local states in this situation are favourable for the agents.

`ALGORITHM`

The algorithm is based on verification of global situations from $Score_{Ag}(t)$ with the local set of goals of agents and constraints. The problem is described by the formula f:

$$f = \bigwedge_{ag \in Ag} t(ag) \wedge f_C \wedge f_\varphi$$

where $t(ag)$ is the disjunction of targets of the agent ag, and $f_C \wedge f_\varphi$ is the formula investigated in Section 5.1 representing all admissible situations without the global conflict regarding the threshold t.

Situations, which can be found using this algorithm, are better then the previous one – local preferences are taken into account.

Example 11. The formula $f_C \wedge f_\varphi$ has been already constructed in the previous example. Formulas $t(ag_1)$ and $t(ag_2)$ are based on sets of goals of agents ag_1 and ag_2, respectively. Example 4 shows the way the formula $t(ag_1)= a_2b_2 \vee a_2b_1$ can be found. The formula $t(ag_2)$ is found in the same way: $t(ag_2)= l_2$.

Thus, the formula f is the following conjunction:

$f = (a_2b_2 \vee a_2b_1) \wedge l_2 \wedge (a_1b_1k_2l_2m_0 \vee a_1b_1k_2l_2m_1 \vee a_1b_1k_2l_2m_2 \vee a_1b_2k_2l_2m_1 \vee a_1b_2k_2l_2m_2 \vee a_1b_0k_2l_2m_1 \vee a_2b_0k_2l_1m_0 \vee a_2b_0k_2l_1m_1 \vee a_2b_1k_2l_1m_1 \vee a_2b_0k_2l_1m_2 \vee a_2b_1k_2l_1m_2 \vee a_2b_2k_2l_1m_2 \vee a_2b_0k_1l_0m_2 \vee a_2b_1k_1l_0m_2 \vee a_2b_2k_1l_0m_2 \vee a_2b_0k_2l_0m_2 \vee a_2b_1k_2l_0m_2 \vee a_2b_2k_2l_0m_2 \vee a_1b_1k_0l_0m_1 \vee a_1b_1k_1l_0m_1 \vee a_1b_1k_1l_1m_1 \vee a_1b_1k_2l_0m_1 \vee a_1b_1k_2l_1m_1 \vee a_1b_0k_2l_0m_1 \vee a_1b_2k_2l_0m_1 \vee a_1b_0k_2l_1m_1 \vee a_1b_2k_2l_1m_1 \vee a_1b_2k_2l_1m_2)$

Within the given data no solution for this problem can be found - the goals of the agents cannot coexist, so they are rejected by the constraints. We will look for the solution in the closeness neighbourhood of the local targets (in the local closeness).

5.3 Consensus on local closeness

`INPUT`

The system with constraints M_{Ag} defined in Section 3.
t - an acceptable threshold of the objective global conflict for Ag.
The closeness threshold α.

`OUTPUT`

All situations with the objective evaluation reduced to degree at most t, and without local conflict for any agent. The new situation can be constructed from the local states closeness, i.e., from the states having the distance from those in the information table less than α.

`ALGORITHM`

The algorithm is similar to the previous one, but each state from the set of goals of any agent is enlarged on the closeness. Precisely, the boolean formula f' defines the solution.

$$f' = \bigwedge_{ag \in Ag} t'(ag) \wedge f_C \wedge f_\varphi$$

where $t'(ag)$ is the formula which describes the agent ag's set of targets with closeness, that is each state from the set of targets and state closeness are considered. The formula f_C describes global situations from the set $Score_{Ag}(t)$, and f_φ stands for constraints.

Example 12. The formula f_φ has been defined in Example 6, and f_C in Example 7.

Let us consider closeness of the goals with the threshold $\alpha = \frac{1}{2}$, i.e. if $d(s, s') < \frac{1}{2}$, then a local state s' is close to s. Let the distance function be defined by

$$d(s, s\prime) = \frac{1}{card(A(ag))} \sum_{a \in A(ag)} |s(a) - s(a')|$$

For the agent ag_1, the closeness neighbourhood with the threshold $\frac{1}{2}$ is not giving new states. Thus, the formula for the local goals of this agent remains the same as one found in Example 4. The local set of goals for the agent ag_2 is shown in Table 8.

states	k	l	m	decision	order e_{ag2}
s1	2	2	2	1	1
$s1_1$	2	1	2	1	
$s1_2$	2	2	1	1	
s2	1	2	2	1	$\frac{2}{3}$
$s2_1$	0	2	2	1	
s3	1	1	2	0	$\frac{1}{3}$
$s3_1$	0	1	2	0	
$s3_2$	1	0	2	0	
s4	1	1	1	0	$\frac{1}{3}$
$s4_1$	0	1	1	0	
$s4_2$	1	0	1	0	
$s4_3$	1	1	0	0	
s5	2	0	1	0	0
$s5_1$	2	1	1	0	
$s5_2$	2	0	2	0	
$s5_3$	2	0	0	0	

Table 8. Local states of agent ag_2 with closeness

One can notice that the states from the closeness neighbourhood of s_1 can be the same as those from the closeness neighbourhood of the state s_5 while the states s_1 and s_5 have completely different evaluation values. We will take the upper boundary of the set of target states (as specified in Table 8 by the states with decision 1).

In order to find out the minimal rules for decision 1, the discernibility between the set of local goals and the other states has to be found. The discernibility matrix is presented in Table 9.

	s1	s1$_1$	s1$_2$	s2	s2$_1$
s3	k, l	k	k, l	l	k, l
s3$_1$	k, l	k	k, l, m	k, l	l
s3$_2$	k, l	k, l	k, l, m	l	k, l
s4	k, l, m	k, m	k, l	l, m	k, l, m
s4$_1$	k, l, m	k, m	k, l	k, l, m	l, m
s4$_2$	k, l, m	k, l, m	k, l	l, m	k, l, m
s4$_3$	k, l, m	k, m	k, l, m	l, m	k, l, m
s5	l, m	l, m	l	k, l, m	k, l, m
s5$_1$	l, m	m	l	k, l, m	k, l, m
s5$_2$	l	l	l, m	k, l	k, l
s5$_3$	l, m	l, m	l, m	k, l, m	k, l, m

Table 9. Discerni ility matrix

Prime implicants for each considered state are as follows: $s_1 : l$, s_{1_1}: $k \wedge l \wedge m$, s_{1_2}: l, $_2$: l and s_{2_1}: l. These attributes are considered while generating the decision rules and consequently $t'(ag_2)$ is the formula as follows. We are always looking for the minimal rules to simplify the formula and speed up the computation.

$t'(ag_2) = l_2 \vee k_2 l_1 m_2 \vee l_2 \vee l_2 \vee l_2 = l_2 \vee k_2 l_1 m_2$ where

$t'(ag_1)$ has been found in Example 4, i.e., $t'(ag_1) = a_2 b_2 \vee a_2 b_1$. Thus the formula f' is as follows:

$f\prime = (a_2b_2 \vee a_2b_1) \wedge (l_2 \vee k_2l_1m_2) \wedge (a_1b_1k_2l_2m_0 \vee a_1b_1k_2l_2m_1 \vee a_1b_1k_2l_2m_2 \vee a_1b_2k_2l_2m_1 \vee a_1b_2k_2l_2m_2 \vee a_1b_0k_2l_2m_1 \vee a_2b_0k_2l_1m_0 \vee a_2b_0k_2l_1m_1 \vee a_2b_1k_2l_1m_1 \vee a_2b_0k_2l_1m_2 \vee a_2b_1k_2l_1m_2 \vee a_2b_2k_2l_1m_2 \vee a_2b_0k_1l_0m_2 \vee a_2b_1k_1l_0m_2 \vee a_2b_2k_1l_0m_2 \vee a_2b_0k_2l_0m_2 \vee a_2b_1k_2l_0m_2 \vee a_2b_2k_2l_0m_2 \vee a_1b_1k_0l_0m_1 \vee a_1b_1k_1l_0m_1 \vee a_1b_1k_1l_1m_1 \vee a_1b_1k_2l_0m_1 \vee a_1b_1k_2l_1m_1 \vee a_1b_0k_2l_0m_1 \vee a_1b_2k_2l_0m_1 \vee a_1b_0k_2l_1m_1 \vee a_1b_2k_2l_1m_1 \vee a_1b_2k_2l_1m_2)$

After reduction, we get:

$f' = a_2b_1k_2l_1m_2 \vee a_2b_2k_2l_1m_2$

Thus, situations presented in Table 10 are proposed as the solution in the conflict i.e. the consensus.

situation	a		k	l	m
S1	2	1	2	1	2
S2	2	2	2	1	2

Table 10. Not conflicting situations

5.4 Consensus based on acceptable situations

The consensus problem based on acceptable situations can be defined by:

`INPUT`

The system with constraints M_{Ag} defined in Section 3.
h – the acceptable level of the global preference function.

`OUTPUT`

The set of all acceptable situations with the threshold h i.e. for which the global preference function is greater than or equal h.

`ALGORITHM`

Our algorithm requires generating all admissible situations and scoring them by applying the global preference function. Then the set $S_{accept}(h)$ can easily be found and described by the left side of an appropriate decision formulas like in theconsensus problem.

Another possible solution is to distribute the required threshold into every agent: $h \leq h_1 + ... + h_n$. Then for any agent, the description of local states satisfying h_i has to be found (formula $t(ag_i)$). The conjunction of components found in the previous step constructs the boolean formula, whose prime implicants form the solutions (non-conflicting situations). However the formula has to be verified against the constraints. For any different distribution the new formula must be created e.g. $f_1, ..., f_m$. The disjunction of these formulas $f = f_1 \vee ... \vee f_m$ describes the problem of finding all possible non-conflicting situations. Summarising, the whole formula is as follows:

$$f = \bigvee_{1 \leq i \leq m} f_i$$

where m is the number of possible distributions of the threshold h and f_i is is the formula describing the consensus problem in the i–th distribution:

$$f_i = \bigwedge_{ag \in Ag} t_i(ag) \wedge f_\varphi$$

where $t_i(ag)$ is the formula which describes the agent ag's set of targets with the given threshold in the i–th distribution and f_φ specifies the constraints.

Conflict level distribution The distribution of the global conflict level consists in passing the conflict from the global into the local level. The way of dividing the global conflict level should reflect the global preference function used in the given conflict. That is all agents features (e.g. agent importance) applied in the global preference function must be considered. Here in the paper both simple global preference function and the way of distribution are used (see Section 6).

5.5 Consensus considering both agents preferences and the global good

The consensus problem defined in the previous section is enlarged with the requirement to consider the global good too i.e.:

`INPUT`

The system with constraints M_{Ag} defined in Section 3.
t – an acceptable threshold of the objective global conflict for Ag.
h – the acceptable level of the global preference function.

`OUTPUT`

All situations with the objective evaluation reduced to degree at most t, and belonging to the set $S_{accept}(h)$, i.e., for which the global preference function is greater than or equal h.

`ALGORITHM`

The first step of the algorithm requires calculating prime implicants of the boolean formula described in the previous section. The next step is to verify the resulting formula from the first step with the formula describing the global situations from $Score_{Ag}(t)$. The way the formula describing the $Score_{Ag}(t)$ is created was shown in Example 7.

6 Example

The simple example of three co–operative intelligent agents is presented in this section. The example recalls the previously defined notions in another type of situation and exemplifies algorithms for resolving the consensus problem based on agents preferences Problem 5.4 and 5.5.

6.1 Conflict subject

Agents ag_1, ag_2 and ag_3 have to paint their elements of the car with the colours accordingly: sea-green, violet and coral. The appropriate colours can be obtained by mixing red, green and blue components as follows. Sea-green colour can be obtained from 2 bottles of blue colour and 1 bottle of green, violet by mixing 2 bottles of blue, 2 bottles of red and one bottle of green and coral from 2 bottles of red and 2 of green. However there are only 4 bottles of red, 4 bottles of blue and 3 of green colour and a single bottle cannot be divided or shared between the agents. The deal is to paint the elements as best as possible i.e. with the appropriate colour or the closest possible shade.

Let us consider the following attributes:

ag_1: v, b (Italian agent: "verde", "blue");

ag_2: c, z, n (Polish agent: "czerwony", "zielony", "niebieski");

ag_3: r, g (English agent: "red", "green")

where r, c denote the number of bottles with red component taken by agents ag_2 and ag_3, respectively. Similarly, g, z, v denote the number of taken bottles with the green component and b, n with the blue one, respectively.

Local states of agents are presented in Table 11, Table 12 and Table 13.

v		e_{ag1}
≥ 1	≥ 2	1 (see-green)
≥ 1	1	$\frac{1}{2}$
0	≥ 1	$\frac{1}{4}$
≥ 0	0	0

Table 11. Agent ag_1's local states

c	z	n	e_{ag2}
≥ 2	≥ 1	≥ 2	1 (violet)
≥ 1	0	≥ 1	$\frac{3}{4}$(light shade)
1	1	1	$\frac{3}{4}$(dark shade)
≥ 1	≥ 1	0	$\frac{1}{2}$
0	≥ 1	≥ 1	$\frac{1}{2}$
0	≥ 1	0	0
≥ 1	0	0	0
0	0	≥ 1	0
0	0	0	0

Table 12. Agent ag_2's local states

States with a condition on attribute values (e.g. $\geq$) denote all states with values in the Cartesian product of attributes domains restricted to the given condition. Thus for example the state $c \geq 1$ and $z \geq 1$ and $n \geq 1$ represents all the states with values from the set $E = \{2,3,4\} \times \{1,2,3\} \times \{2,3,4\}$. The condition *greater then* comes from the assumption that agents can take from the stock more bottles then they really need.

r	g	e_{ag3}
≥2	≥2	1 (coral)
≥2	1	$\frac{3}{4}$(orange)
1	≥1	$\frac{1}{2}$
≥1	0	0
0	≥1	0
0	0	0

Table 13. Agent ag_3's local states

Sets of targets for each agent are seperated with double lines (the boundary level is $\frac{1}{2}$).

The constraints are due to the limited number of bottles with components i.e., $r + c \leq 4$, $g + z + v \leq 3$, $b + n \leq 4$. They can be converted into the propositional formula:

$f_\varphi = (r_0c_0 \vee r_0c_1 \vee r_0\ c_2 \vee r_0c_3 \vee r_0c_4 \vee r_1c_0 \vee r_1c_1 \vee r_1c_2 \vee r_1c_3 \vee r_2c_0 \vee r_2c_1 \vee r_2c_2 \vee r_3c_0 \vee r_3c_1 \vee r_4c_0) \wedge (g_0z_0v_3 \vee g_0z_0v_2 \vee g_0z_0v_1 \vee {}_0z_0v_0 \vee g_0z_3 v_0 \vee g_0z_2v_0 \vee g_0z_1v_0 \vee g_3z_0v_0 \vee g_2z_0v_0 \vee g_1z_0v_0 \vee g_0z_1v_2 \vee g_0z_2v_1 \vee g_1z_0v_2 \vee g_2z_0v_1 \vee g_1z_2v_0 \vee g_2z_1v_0 \vee g_0z_1v_1 \vee g_1z_0v_1 \vee g_1z_1v_0 \vee g_1z_1v_1\ ) \wedge (b_0n_0 \vee b_0n_1 \vee b_0n_2 \vee b_0n_3 \vee b_0n_4 \vee b_1n_0 \vee b_1n_1 \vee b_1n_2 \vee b_1n_3 \vee b_2n_0 \vee b_2n_1 \vee b_2n_2 \vee b_3n_0 \vee b_3n_1 \vee b_4n_0)$

The current global situation (conflicting) is presented in Table 14.

	v		c	z	n	r	g
Sc	1	2	0	2	2	4	0

Table 14. The current glo al situation

6.2 Analysis

The value of the global preference function for the current situation presented in 14 is

$$p(Sc) = \sum_{i=1}^{3} e_{ag_i}(s_i) = 1 + \frac{1}{2} + 0 = \frac{3}{2}$$

where s_i is the agent ag_i's part of the situation Sc.

Let the threshold h be $2\frac{3}{4}$. Thus $Cp(Sc) = 2\frac{3}{4} - 1\frac{1}{2} = 1\frac{1}{4}$ and agents are in conflict.

Concerning the specified states evaluation we can distribute this threshold between agents in the following manner:

$(h_1(ag_1) = 1, h_1(ag_2) = 1, h_1(ag_3) = 1) \wedge ((h_2(ag_1) = 1, h_2(ag_2) = \frac{3}{4},$
$h_2(ag_3) = 1) \vee (h_2(ag_1) = 1, h_2(ag_2) = 1, h_2(ag_3) = \frac{3}{4}))$

where $h_i(ag)$ is a threshold in the i–th distribution for ag.

For each agent we have to find the formulas describing the states locally evaluated into 1. Additionally the formula describing the states evaluated by agents ag_2 and ag_3 into $\frac{3}{4}$ has to be found. The formulas are the left-hand sidse of minimal rules found from the agents' local tables. The decision is 1 for the states with score equal to the given threshold h. We obtained the following formulas:

$t_1(ag_1) = t_2(ag_1) = t_3(ag_1) = v_3b_4 \vee v_3b_3 \vee v_3b_2 \vee v_2b_4 \vee v_2b_3 \vee v_2b_2 \vee v_1b_4 \vee v_1b_3 \vee v_1b_2$

$t_1(ag_2) = t_3(ag_2) = c_4z_3n_4 \vee c_4z_3n_3 \vee c_4z_3n_2 \vee c_4z_2n_4 \vee c_4z_2n_3 \vee c_4z_2n_2 \vee c_4z_1n_4 \vee c_4z_1n_3 \vee c_4z_1n_2 \vee c_3z_3n_4 \vee c_3z_3n_3 \vee c_3z_3n_2 \vee c_3z_2n_4 \vee c_3z_2n_3 \vee c_3z_2n_2 \vee c_3z_1n_4 \vee c_3z_1n_3 \vee c_3z_1n_2 \vee c_2z_3n_4 \vee c_2z_3n_3 \vee c_2z_3n_2 \vee c_2z_2n_4 \vee c_2z_2n_3 \vee c_2z_2n_2 \vee c_2z_1n_4 \vee c_2z_1n_3 \vee c2z_1n_2$

$t_2(ag_2) = c_4z_0n_4 \vee c_4z_0n_3 \vee c_4z_0n_2 \vee c_3z_0n_4 \vee c_3z_0n_3 \vee c_3z_0n_2 \vee c_2z_0n_4 \vee c_2z_0n_3 \vee c_2z_0n_2 \vee c_4n_1 \vee c_3n_1 \vee c_2n_1 \vee c_1n_4 \vee c_1n_3 \vee c_1n_2 \vee c1n_1$

$t_1(ag_3) = t_2(ag_3) = r_4g_3 \vee r_3g_3 \vee r_2g_3 \vee r_4g_2 \vee r_3g_2 \vee r_2g_2$

$t_3(ag_3) = r_4g_1 \vee r_3g_1 \vee r_2g_1$

Thus the problem can be transformed into the problem of finding prime implicants of the following boolean formula f, where f_φ has been presented in Section 6.1.

$f = (t_1(ag_1) \wedge t_1(ag_2) \wedge t_1(ag_3) \wedge f_\varphi) \vee (t_2(ag_1) \wedge t_2(ag_2) \wedge t_2(ag_3) \wedge f_\varphi) \vee (t_3(ag_1) \wedge t_3(ag_2) \wedge t_3(ag_3) \wedge f_\varphi)$

Finally, prime implicants of the formula f form the solution i.e. non–conflicting situations. Table 15 presents all non-conflicting situations, where S1 comes from the second distribution and S2-S10 from the third.

	v		c	z	n	r	g
S1	1	2	2	1	2	2	1
S2	1	2	1	0	2	2	2
S3	1	3	1	0	1	2	2
S4	1	2	1	0	1	2	2
S5	1	3	2	0	1	2	2
S6	1	2	2	0	1	2	2
S7	1	2	1	0	2	3	2
S8	1	3	1	0	1	3	2
S9	1	2	1	0	1	3	2
S10	1	2	2	0	2	2	2

Table 15. Non-conflicting situation

7 Coalitions

Coalitions can be extracted by finding the relations among agents in current and/or historical data. Roughly speaking agents are in a coalition when their state evaluation on the same situation is similar (with respect to the given threshold). More precisely two agents coalition is a tolerance relation γ such that:

$$< ag, ag\prime > \in \gamma \Leftrightarrow D(ag, ag\prime) \leq \ t$$

where t is the given threshold and D is the distance function defined as follows in general:

$$D(ag, ag\prime) = F(f(e_{ag1}(S1), e_{ag2}(S1)), ..., f(e_{ag1}(Sm), e_{ag2}(Sm)))$$

where $e_{ag}(S)$ is agent's ag evaluation of the state from the situation S, m – the number of situations available, f and F chosen, suitable functions e.g.

$$D(ag, ag\prime) = \frac{1}{m} \sum_{i=1}^{m} |e_{ag}(S_i) - e_{ag'}(S_i)|$$

A coalition C is the set of agents such that

$$\forall (ag, ag\prime \in C) max D(ag, ag\prime) \leq \ t$$

Example 13. Let us consider five agents and 8 situations. We are going to find coalitions among the agents with the threshold t=7/8. In the table below (Table 16) we present only the evaluation of each agent state in the considered situation.

	e_{ag1}	e_{ag2}	e_{ag3}	e_{ag4}	e_{ag5}
S1	1	0	2	1	0
S2	0	0	1	2	1
S3	2	0	2	1	0
S4	0	1	2	2	2
S5	0	0	2	1	0
S6	1	1	2	1	0
S7	0	1	0	2	0
S8	1	2	1	2	1

Table 16. Local states evaluations of exemplary data

Now distances between agents must be calculated e.g.:

$$D(ag_1, ag_2) = \frac{1}{8}(|1-0|+|0-0|+|2-0|+|0-1|+|0-0|+|1-1|+|0-1|+|1-2|) = \frac{3}{4}$$

The distances can be presented in the distance table - Table 17.

Thus only one bigger coalition can be determined: $\{ag_1, ag_2, ag_5\}$.

	ag_1	ag_2	ag_3	ag_4	ag_5
ag_1					
ag_2	$\frac{6}{8}$				
ag_3	$\frac{7}{8}$	$\frac{11}{8}$			
ag_4	$\frac{9}{8}$	$\frac{6}{8}$	$\frac{8}{8}$		
ag_5	$\frac{7}{8}$	$\frac{5}{8}$	$\frac{8}{8}$	$\frac{8}{8}$	

Table 17. The distance ta le

8 Calculation strategies

Boolean calculations of formulas described in the previous section can be time consuming. In the consensus problem we have to verify the local goals $f_1, ..., f_n$ against the formula of admissible situations and/or constraints f. This usually yields long formulas looking like : $g = f_1 \wedge f_2 \wedge ... \wedge f_n \wedge f$. Calculating prime implicants of such formulas is usually a hard-computational problem. Therefore depending on the formula, some simple strategies or eventually quite complex heuristics must be used to resolve the problem in real time. The important fact which can be used in calculation strategies is that the result (if any) is the disjunction of selected components of the formula f. This last remark has been applied in the program module created by the author.

The discussed problems, especially consensus problem, can be treated as the numerical CSP problems as well (see e.g. [3, 4] or [32]). The entry points are the constraints, which in this case will not be transformed into the propositional form. The quality score of the global situation must be set to the already computed situation. If the score does not satisfy the threshold, a next solution has to be searched.

8.1 Simple strategies

Simple strategies can be based on the Boolean algebra rules. First, the absorption rule has to be considered when choosing the formulas to calculate the formulas conjunction – a shorter formula can strongly reduce the longer formula being an extension of the shorter one. In the case when the attribute domain of a given component is small, it may be worthy to replace that component with the disjunction of negations using de Morgan rules. Let us take for example the formula $f = a_0b_2 \vee a_0b_0 \vee a_0b_1 \vee a_0c_1$. Assuming $\overline{b_2} = b_0 \vee b_1$ the disjunction $a_0b_0 \vee a_0b_1 = a_0(b_0 \vee b_1)$ can be replaced with $a_0\overline{b_2}$. Thus we obtain $f = a_0b_2 \vee a_0\overline{b_2} \vee a_0c_1$ which can be reduced further using the absorption rule to $f = a_0$.

Example 14. Let us present an example of a simple strategy with the consensus problem. The problem is to find all prime implicants of the boolean formula $f_1 \wedge f_2 \wedge ... \wedge f_n \wedge f$. To make the calculations faster the shortest formula from $f_1, ..., f_n$ (e.g., f_1) should be chosen and matched against the formula f. The formula f is then reduced to the components satisfying the formula f_1. Next, the algorithm repeats looking for the shortest formula among f_2,...,f_n. The process ends when the whole formula f is eliminated (no solution can be found) or we have scanned through all the components of formulas $f_1, ..., f_n$ - all prime implicants have been found.

Let us consider the calculation from Example 12. There are formulas $f_1 = a_2b_2 \vee a_2b_1$, $f_2 = l_2 \vee k_2l_1m_2$ and a formula that describes these admissible situations for which the quality value is greater than the given threshold, i.e.,

$f = a_1b_1k_2l_2m_0 \vee a_1b_1k_2l_2m_1 \vee a_1b_1k_2l_2m_2 \vee a_1b_2k_2l_2m_1 \vee a_1b_2k_2l_2m_2 \vee a_1b_0k_2l_2m_1 \vee a_2b_0k_2l_1m_0 \vee a_2b_0k_2l_1m_1 \vee a_2b_1k_2l_1m_1 \vee a_2b_0k_2l_1m_2 \vee a_2b_1k_2l_1m_2 \vee a_2b_2k_2l_1m_2 \vee a_2b_0k_1l_0m_2 \vee a_2b_1k_1l_0m_2 \vee a_2b_2k_1l_0m_2 \vee a_2b_0k_2l_0m_2 \vee a_2b_1k_2l_0m_2 \vee a_2b_2k_2l_0m_2 \vee a_1b_1k_0l_0m_1 \vee a_1b_1k_1l_0m_1 \vee a_1b_1k_1l_1m_1 \vee a_1b_1k_2l_0m_1 \vee a_1b_1k_2l_1m_1 \vee a_1b_0k_2l_0m_1 \vee a_1b_2k_2l_0m_1 \vee a_1b_0k_2l_1m_1 \vee a_1b_2k_2l_1m_1 \vee a_1b_2k_2l_1m_2$.

According to our strategy the formula f_1 is chosen. It simplifies the formula f to $f' = a_2b_1k_2l_1m_1 \vee a_2b_1k_2l_1m_2 \vee a_2b_2k_2l_1m_2 \vee a_2b_1k_1l_0m_2 \vee a_2b_2k_1l_0m_2 \vee a_2b_1k_2l_0m_2 \vee a_2b_2k_2l_0m_2$. Then the formula f_2 is applied and the result is obtained i.e. $f'' = a_2b_1k_2l_1m_2 \vee a_2b_2k_2l_1m_2$.

Another way of calculation, when looking for the best solution, is to choose the components of the formula f relative to the global evaluation (starting from the best score) and verify this component within the formulas $f_1, ..., f_n$.

8.2 Agent clustering

The detection of coalitions can help us in resolving the consensus problem. Problems with many agents involved can be divided into smaller problems and solved separately. Furthermore, we can apply agents clustering based on the coalitions (for basics about cluster analysis see e.g. [10]). The hypothesis is that we can rid of some of the agents by replacing each coalition with one representative, and resolve equivalently the consensus problem between the new set of agents. (We permit the boundary level of information lost.) The proposals described above require redefining the conflict situation (including constraints). Another approach is to take advantage of coalition knowledge during calculation of consensus problem. This approach consists of the following steps:

1. Finding coalitions.
2. Choosing one agent from the coalition as the coalition representative.
3. Removing from the formula $g = f_1 \wedge f_2 \wedge ... \wedge f_n \wedge$f (describing consensus problem) all formulas f_i of agents removed by clustering.
4. Calculating prime implicants of the new formula g'.
5. (Optional). Verifying formula g' within formulas f_i removed in the third step.

After the forth step we obtain the approximated solution. The local goals of agents reduced by clustering are not considered. Thus the 5th step is recommended to reach full solution. The algorithm presented here (even including the 5th point) usually reduces the number of conjunctions needed for finding the solution. This is due to reducing the longest formula f (representing constraints) first by most significant formulas. This reduction has also been proven experimentally. The calculation of all prime implicants (4400) of the constraints in Example 6 takes about 7 seconds (within the author program) while the calculation of longer formula resolving consensus problem of the same example takes about 1 second. In the last case some prime implicants of the constraints are not calculated at all – they are reduced by formulas of agents preferences.

Computer implementations of this approach can be reduced by applying the priority among agents. The formulas are matched against constraints in the priority order.

9 Agents moves

The conflict situation is usually flowing (unstable). Agents are changing their views on some issues in respond to changing the external situation and/or other agents' moves. The agents' willingness of changing the state (particularly of giving up some of resources) is the fundamental assumption in the negotiation process. On the other hand the strategy of any agent is to reach the preferred state i.e., the one from its target. Because of constraints, moving from one state to another can cause the other agents to change their states.

Considering acceptable situations, there are two general possibilities when an agent is going to change its state:

1. Improving one agent's state (by changing its current state to the one from the set of targets) does not force other agents into the states out from their set of targets. These moves avoid conflicts.
2. Improving one agent's state causes other agents (agent) to change their states into less preferred.

9.1 Tension

Transition relation for ag is a binary relation μ in $U_{ag} \times U_{ag}$ such that $< s, s\prime > \in \mu$ if s is the current state and s' is any state from the local states and $s \neq s\prime$. *Upward transition* H_{ag} is a relation in $U_{ag} \times U_{ag}$ such that:

$$< s, s\prime > \in H_{ag} \leftrightarrow e_{ag}(s\prime) > e_{ag}(s)$$

and $s, s' \in U_{ag}$. Accordingly, D_{ag} is the *downward transition* relation in $U_{ag} \times U_{ag}$:

$$< s, s\prime > \in D_{ag} \leftrightarrow e_{ag}(s\prime) < e_{ag}(s)$$

In a global situation S, where ag_1 is in the state s_{ag_1} and ag_2 is in s_{ag_2}, there is a tension between ag_1 and ag_2 if the move of ag_1 according to H_{ag_1} requires – due to constraints – the agent ag_2 move to s_{ag_2}' such that $(s_{ag_2}, s_{ag_2}\prime) \in D_{ag_2}$.

Example 15. Let us consider the example from Section 3.

	v		c	z	n	r	g	p
Sc	1	2	0	2	2	4	0	$1\frac{1}{2}$

Table 18. Conflicting situation

Assuming that in the situation from Table 18 the agent ag_3 is going to change its state into $r = 4, g = 1$, this move is only possible due to constraints when the agent ag_1 or ag_2 returns occupied resources i.e. $v = 0$ or $z = 0$. One can notice that releasing resources by ag_1 causes its state to become less preferred, while ag_2 stays in the same state evaluated . Thus in the sense of our definition, ag_3 is in tension with ag_1, while not in the tension with ag_2.

10 Conclusions

We have presented and discussed the extension of the Pawlak conflict model. The understanding of the underlying local states as well as constraints in the given situation is the basis for any analysis of our world. The local preferences as well as the evaluation of the global situation are observed as factors defining the strength of the conflict and can suggest the way to reach the consensus.

The fundamental consensus problem has been analysed in the paper. Then, Boolean reasoning has been successfully applied as a tool for solving presented problems. A program module created by the author allows to resolve much more complex conflicts than presented in the paper.

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