# From Fuzzy Sets to Rough Sets and Back

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## Information–Based Representations of Vagueness

In everyday life, most common concepts expressed by words in natural language are inherently vague [9]. There are two information–based proposals to model it, Zadeh's *fuzzy set theory* [11] and Pawlak's *rough set theory* [6]. Both of them make opaque the sharp boundaries of sets, but their representations regarding vagueness are complementary to each other.

Fuzzy set theory addresses to represent inherent vague aspect of concepts and so this approach is *a priori* by nature. This theory can be built up in two equivalent forms, either by fuzzy membership functions or by $\alpha$-level sets. They express gradual transitions from membership to nonmembership, therefore fuzzy sets fundamentally differ from classical sets.

Rough set theory remains within classical set theory. At the very beginning, the available *primary knowledge* about the objects of interest is represented by a beforehand predefined and definitely detached subset family of the universe (equivalence classes in the standard case). It is called the *base system*, its members are the *base sets*. Setting out from base sets so–called *definable sets* are obtained by set operations (union of equivalence classes in the standard case). Then, lower and upper approximations of sets are formed with the help of definable sets. A reference set is *rough* if the difference of its upper and lower approximations is not empty. In this scheme, these boundary zones around sets represent the vagueness. Clearly, this approach is *a posteriori* by nature.

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A number of studies deal with the relationship between rough set theory and fuzzy set theory. They are usually based on functional approach. There are many opportunities to combine them (rough fuzzy sets, fuzzy rough sets [3]). Recently, a complex computer program package "RoughSets" written mainly in the R language has been come out under GNU General Public License[1] [8].

Vagueness may be considered as a "gradual transition" or "uncertainty". Fuzzy set theory is successfully able to handle vagueness as gradual transition in such a way that it endows sets with *smooth boundaries*.

[1] http://sci2s.ugr.es/dicits/index.php?p=software

Uncertainty is attached to the beforehand given background knowledge about the objects of the universe. This knowledge is necessarily incomplete which is manifested in the fact that some objects are *indiscernible*. Rough set theory assembles such objects into homogeneous blocks, the granules of knowledge, and handles vagueness as uncertainty by the help of them.

A natural question is how a relationship can be built up between *a priori* vagueness managed by fuzzy set theory and *a posteriori* vagueness managed by rough set theory. In rough set theory, a special class of fuzzy sets can be defined in a natural way. It is based on the classical rough membership function which was explicitly introduced by Pawlak and Skowron [7]. It is generalized for different classes of rough set models relying on arbitrary binary relations, total or partial covering of the universe, see, e.g., [2,10]. On the other hand, a fuzzy set can be explained by Frege's Context Principle [4,5]. Its interpretation depends on the context which can be set up, for instance, by background knowledge.

In the study, first, a very general definition of approximation space will be given [1]. In this general context, some possible transitions between fuzzy sets and rough sets and back will be investigated.

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