Generalized Quantifiers with Rough Set Semantics

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Presence of generalized quantifiers, in communicating the semantic meaning and expressiveness of a language is inevitable. There are a handful of approaches dealing with the meaning of generalized quantifiers As the main essence of linguistic quantifier lies in its inherent vagueness, fuzzy set theoreticians came up with different solutions to formalize the meaning of different types of linguistic quantifiers. Semantics of generalized quantifiers in the context of rough set theory, in contrast, yet has not been developed much. In this paper, our aim is to propose a general set-up for interpreting a class of linguistic quantifiers from the perspective of rough sets.

Let us consider the interpretations of some crisp quantifiers in the context of classical set theory. Let $\forall_x^U P$, $\exists_x^U P$, and $N_x^U P$ represent the sentences 'for all x over the domain U, P holds', 'for some x over U, P holds', and 'for none of the x over U, P holds'. The interpretations of the above quantified sentences are respectively, 'every member of U is an element of $I(p)$', 'some member of U are elements of $I(P)$, and 'no member of U is an element of $I(P)$, where $I(P) \subseteq U$ is the interpretation of P. That is, membership of an element to a set plays a crucial role here. Classical set theory and fuzzy set theory are based on membership of an element to a set. Rough set theory, on the other hand, does not talk about belongingness of a single element to a concept; it talks about belongingness of a cluster/block/neighbourhood of elements to a concept. So, in order to propose semantics for quantified sentences in the context of rough set theory some modifications are required. We shall introduce a *notion of covering* for a rough set theoretic representation of a set $X \subseteq U$ in an approximation space (U,R). Thus, the interpretation of a quantified sentence would be shifted from the notion of 'membership to a concept' to the notion of 'covering of a concept'. The main challenges for evaluating a quantified sentence viz., QP_1P_2 where Q denotes the quantifier, and P_1, P_2 denote respectively the restriction and the argument of Q, in the framework of rough set theory are as follows.

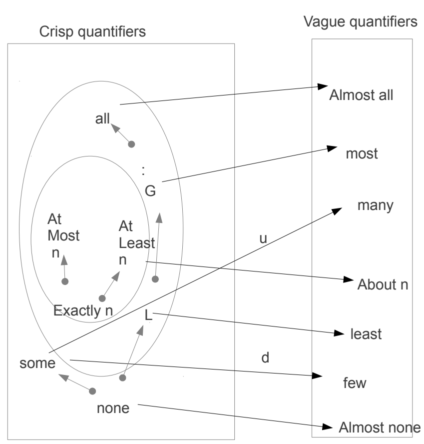
(i) To introduce a rough membership function which can attach finer distinction apart from assigning 1 to the core, $\frac{1}{2}$ to the boundary, and 0 to the outer region of a concept.
(ii) To devise a general mechanism for evaluating a quantified sentence, with generalized linguistic quantifiers, when the vagueness of the restriction and the argument of the concerned quantifier is interpreted in terms of rough sets.

In connection to (i) we shall start from an approximation space (U,R_A) where the equivalence relation R_A is formed based on a set of attributes A. While partitioning the universe U by a number of blocks we would concentrate on a set of attributes A. If a larger set of attributes, say $B(\supseteq A)$, is considered then some finer distinctions among the elements lying in the boundary zone might become visible. The distinctions which become noticeable due to taking into account some addtional attributes may not be so certain that two elements can either be distinguishable or indistinguishable; rather they could yield a degree of variation, or in other words a degree of similarity. There could be another case when the appearance of new objects is considered; i.e. for U' ($U \subseteq U'$), we may need to decide the membership of the new elements to a concept with the help of the available approximation space (U, R_A). To address both the contexts we would introduce a definition of rough membership function for a fuzzy approximation space (U', R_A, Sim), where $U \subseteq U'$ and Sim is a binary fuzzy similarity relation over U'.

In connection to (ii) we first present a value computation scheme in the context of rough set semantics for the sentences of the form $Q_c P_f^1 P_f^2$, i.e., sentences with crisp quantifiers over fuzzy restriction and fuzzy argument. Establishing an assertion that '*for all x the property P holds*', depends on establishing that the domain of interpretation of P coincides with the whole universe of discourse. In contrast, when it is asserted that '*for almost all x the property P holds*' there must be some uncertainty about the choice of the chunk of objects from the universe, for which P holds. To a perceiver, a chunk of objects which may fit to 'almost all', seems to be perceived as similar or close to the chunk representing 'all'. In this paper we propose to see vague quantifiers as blurred/rough images of a set of crisp quantifiers.

Informally, our proposal may be considered as a step towards achieving the diagram presented in Fig.1. The arrows from the set of crisp quantifiers to the set of vague quantifiers represent that which vague quantifier can be obtained as a rough image of which set of crisp quantifiers. Arrows with 'u' and 'd' represent that *many* and *few*, respectively can be obtained as an upward and downward rough image of the quantifier *some*.

Fig.1



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