Rough Granular Computing in Modal Settings: Generalised Approximation Spaces

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Granular computing (GrC) is a methodology or – as ambitious at it sounds – a paradigm in computer science the aim of which is to define and solve computational problems in terms of information granules. These granules are conceived as clumps of objects drawn together due to some criteria such as, e.g., similarity, indiscernibility or functionality. The present paper is concerned with the rough granular computing methodology (RGrC) which is GrC expressed and developed within rough set theory. In this theory the elementary information granules usually are: indiscernibility classes (the classical model), tolerance classes (the tolerance rough set model), or minimal open neighbourhood (the topological model). The main idea which stays behind all these rough set models is to express all pieces of information about a given domain U as some binary relation E between objects belonging to U – e.g.: an equivalence relation, a tolerance relation, and a preorder, respectively. Each relation induces in turn rough approximations of any subset of the universe U. It allows one to express the approximations of sets/concepts in terms of modal operators of some modal logic – e.g.: **S5**, **B**, and **S4**, respectively. Then, given a set of objects $|\alpha|$ which satisfy the formula α, $|\Box\alpha|$ is the interior of $|\alpha|$ (in the corresponding topology or pretopology) and $|\Diamond\alpha|$ is the closure. However, the specific feature of modal logic is the locality: everything is computed starting from a single designated world (called the actual world). Thus a formula α is satisfied by a single world; in other words, formulae may say something only about single objects whereas in GrC one deals with information granules rather than single elements. So, the standard modal approach lacks the expressive power to "*touch*" information granules.

Z. Pawlak and A. Skowron considered a generalisation of approximation spaces defined as a triple (U, I, v), where U is the universe of objects, I is an uncertainty function $I : U \rightarrow \mathcal{P}\mathcal{P}U$ (or, in simplified version, $I : U \rightarrow \mathcal{P}U$), and v is a rough inclusion function $v : \mathcal{P}U \times \mathcal{P}U \rightarrow [0, 1]$ telling to what an extent one set in included in another one. Of course, each function from U to $\mathcal{P}\mathcal{P}U$ may be redefined as a relation R between objects and sets of objects $R \subseteq U \times \mathcal{P}U$; one can also consider its inverse $R^{-1} \subseteq \mathcal{P}U \times U$. In contrast to the standard modal settings, here the granulation of the universe is given explicitly; moreover, both R and R^{-1} may be given GrC based interpretations as *organisation* and *decomposition*.

> *[G]ranulation involves decomposition of whole into parts. Conversely, organisation involves an integration of parts into whole* (L. Zadeh).

When considered from the level of objects the process of granulation of U may be seen as organisation (single objects are organised into larger structures), whereas from the level of the universe it may be interpreted as decomposition (the universe is decomposed into smaller parts). In the paper we take the former standpoint. Thus we shall interpret the relation $R \subseteq U \times \mathcal{P}U$ as passing from objects to granules, that is as *organisation*, whereas the relation $R^{-1} \subseteq \mathcal{P}U \times U$ will be interpreted as passing from granules to objects, that is *decomposition*.

The main idea presented in the paper is to define standard modal operators induced by the relations $R \subseteq U \times \mathcal{P}U$ and $R^{-1} \subseteq \mathcal{P}U \times U$.

A constructive example of I offered by Z. Pawlak and A. Skowron is as follows: let a set F of formulae from some language L describing objects from the universe U. Then $I(x)$ may be defined as a set $\{|\alpha| : \alpha \in F \; \& \; x \in |\alpha|\}$, where $|\alpha|$ is a set of world/objects in which the formula is true. In this case I is a function from U to $\mathcal{P}\mathcal{P}U$. Let R be a relational version of I, that is $R = \{(x, Y) \in U \times \mathcal{P}U : Y \in I(x)\}$.

Now we can feed the standard definitions of modal operators with R; for any α from L we have:

$v_x(\Diamond\alpha) = 1$ if for some $X \in I(x)$ $v_X(\alpha) = 1$, and 0 otherwise,
$v_x(\Box\alpha) = 1$ if for all $X \in I(x)$ $v_X(\alpha) = 1$, and 0 otherwise.

Interestingly, α tells us something about a set (an information granule) X. In the standard modal approach every formula is satisfied by a single world; in the case above α is satisfied by a set of objects/worlds. As noted earlier, the intended interpretation of R is *organisation*: passing from single objects to information granules. In the GrC methodology we need also *decomposition*, which in this case is simply the membership $\ni$, that is $\{(X, x) \in \mathcal{P}U \times U : x \in X\}$. The corresponding modal operators will be denoted by $\Box_\ni$ and $\Diamond_\ni$.

Interestingly, the new modal operators allow one to define the closure and interior operators equal to that based on standard modal frames; better still, they allow one to deal directly with information granules and even to explicitly consider the properties of information granules. Thus, this new modal framework is better suited to express RGrC methodology.

We are going to also discuss the modal counterpart of rough inclusion which return real numbers from $[0, 1]$ and finally consider the simplified case of the uncertainty function I.

References

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