Optimal Decision Rules, Generators and Rough Sets

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Certain and generalized decision rules are typical kinds of rules considered in the rough sets area. In practice, for a given consequent, only decision rules with minimal antecedents are of interest. Such decision rules will be called *optimal*. It was proved in [2] that antecedents of optimal certain and generalized decision rules are *generators*, where a generator is defined as a set *S* of features (e.g. a set of attribute-value pairs) such that the number of objects possessing *S* is different from all cardinalities of sets of objects possessing proper subsets of *S*. Algorithms for finding such optimal decision rules were offered in [2] as well. Their efficiency lies in the fact that antecedents of candidate decision rules are restricted only to generators, the number of which is often by an order or even a few orders of magnitude less than the number of potential antecedents created from sets of conditional features.

A number of modifications of original rough set model has been proposed in the literature (see e.g. [1, 6-8]). Modifications typically consist in redefining approximations of rough sets according to some evaluation formulae, which influences properties of decision rules corresponding to these changes. In this paper, we claim that not only optimal certain and generalized decision rules, but also a wide class of optimal decision rules of other kinds, including those based on modifications of the classical rough set model, have generators as antecedents, which allows skipping the creation of candidate rules with non-generator antecedents in the rules' generation process. In particular, we claim that the property of having a generator as an antecedent holds for optimal decision rules with imposed constraints on minimal acceptable values of any set of *ACBC-evaluation measures*, which we define as follows: An *ACBC-evaluation measure* of a rule $X \rightarrow Y$ is a measure that can be expressed as a formula built from at most the following four components: 1) $P(X)$ - the fraction of objects possessing X, 2) $P(Y)$ - the fraction of objects possessing Y, 3) $P(XY)$ - the fraction of objects possessing both X and Y, and 4) constants. Note that marginal probabilities $P(\bar{X})$, $P(\bar{Y})$ and joint probabilites $P(\bar{X}Y)$, $P(X\bar{Y})$ and $P(\bar{X}\bar{Y})$ are derivable from $P(X)$, $P(Y)$ and $P(XY)$ (e.g. $P(\bar{X}) = 1 - P(X)$, $P(\bar{X}Y) = P(Y) - P(XY)$, $P(\bar{X}\bar{Y}) = P(\bar{X}) - P(\bar{X}Y) = (1 - P(X)) - (P(Y) - P(XY))$. Hence, any measure of a decision rule $X \rightarrow Y$ that is defined in terms of $P(X)$, $P(Y)$, $P(\bar{X})$, $P(\bar{Y})$, $P(XY)$, $P(\bar{X}Y)$, $P(X\bar{Y})$, $P(\bar{X}\bar{Y})$ and eventually constants is an *ACBC*-evaluation measure. In fact, medical experts find that uncovered negative and accuracy, original definitions of which refer to $P(\bar{X}\bar{Y})$, belong to most useful decision rule measures [5]. In Table 1, we provide definitions of example *ACBC*-evaluation measures such as support, confidence, coverage, novelty, certainty factor, dependence factor, cosine, Jaccard, accuracy, uncovered negative (see e.g. [3-5] for analysis of their properties).

Table 1. Example *ACBC*-evaluation measures of rules

measure	definition
support($X \rightarrow Y$)	$P(XY)$
confidence($X \rightarrow Y$)	$\frac{P(XY)}{P(X)}$
coverage($X \rightarrow Y$)	$\frac{P(XY)}{P(Y)}$
novelty($X \rightarrow Y$)	$P(XY) - P(X) \times P(Y)$
lift($X \rightarrow Y$)	$\frac{P(XY)}{P(X) \times P(Y)}$
certaintyFactor($X \rightarrow Y$)	$\begin{cases} \frac{conf(X \rightarrow Y) - P(Y)}{1 - P(Y)} & \text{if } conf(X \rightarrow Y) > P(Y) \\ 0 & \text{if } conf(X \rightarrow Y) = P(Y) \\ -\frac{P(Y) - conf(X \rightarrow Y)}{P(Y) - 0} & \text{if } conf(X \rightarrow Y) < P(Y) \end{cases}$ $= \begin{cases} \frac{P(XY) - P(X) \times P(Y)}{P(X) - P(X) \times P(Y)} & \text{if } P(XY) > P(X) \times P(Y) \\ 0 & \text{if } P(XY) = P(X) \times P(Y) \\ -\frac{P(X) \times P(Y) - P(XY)}{P(X) \times P(Y) - 0} & \text{if } P(XY) < P(X) \times P(Y) \end{cases}$
dependenceFactor($X \rightarrow Y$)	$\begin{cases} \frac{conf(X \rightarrow Y) - P(Y)}{max\_conf(X \rightarrow Y\|_{P(X),P(Y)}) - P(Y)} & \text{if } conf(X \rightarrow Y) > P(Y) \\ 0 & \text{if } conf(X \rightarrow Y) = P(Y) \\ -\frac{P(Y) - conf(X \rightarrow Y)}{P(Y) - min\_conf(X \rightarrow Y\|_{P(X),P(Y)})} & \text{if } conf(X \rightarrow Y) < P(Y) \end{cases}$ $= \begin{cases} \frac{P(XY) - P(X) \times P(Y)}{\min\{P(X), P(Y)\} - P(X) \times P(Y)} & \text{if } P(XY) > P(X) \times P(Y) \\ 0 & \text{if } P(XY) = P(X) \times P(Y) \\ -\frac{P(X) \times P(Y) - P(XY)}{P(X) \times P(Y) - \max\{0, P(X) + P(Y) - 1\}} & \text{if } P(XY) < P(X) \times P(Y) \end{cases}$
Jaccard($X \rightarrow Y$)	$\frac{P(XY)}{P(X) + P(Y) - P(XY)}$
accuracy($X \rightarrow Y$)	$P(XY) + P(\bar{X}\bar{Y}) = 1 + 2P(XY) - P(X) - P(Y)$
uncoveredNegative($X \rightarrow Y$)	$P(\bar{X}\bar{Y}) = 1 - P(X) - P(Y) + P(XY)$.

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