A relational logic for spatial contact based on rough set approximation

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A *Boolean contact algebra* is a Boolean algebra enhanced by a binary relation $\mathscr{C}$ which satisfies several axioms reflecting various properties of contact among regions of a geometrical or topological space, see for example, [1] and the references therein for a comprehensive background. In many cases, however, regions cannot be exactly determined, but only described by approximations; a case in point are the regions of a computer screen determined by the chosen resolution, membership in which can only be described by a lower and an upper approximation. To model our intuition of approximate region we follow the paradigm of rough sets [6] (see also [3]) and suppose that there is a collection B of *crisp* or *definable* regions, which forms a Boolean algebra with natural order $\leq$. The crisp regions delineate the bounds up to the granularity of which other regions can be observed. The power of observation is expressed by pairs of the form $\langle a,b \rangle$, $a \leq b$, where a,b are definable regions. In other words, for each (unknown) region x there is a lower bound $i(x) = a$ and an upper bound $h(x) = b$, both of which are crisp, up to which x is discernible. If $i(x) = h(x)$, then x itself is definable. The pair $\langle i(x), h(x) \rangle$ is called an *approximating region*. We also assume that the bounds $\langle i(x), h(x) \rangle$ are best possible. In [2] we have generalized the notion of a Boolean contact algebra to the class of *approximating contact algebras* (ACAs). The algebraic part of an ACA – denoted by AA – is definitionally equivalent to a regular double Stone algebra (as was to be expected). Regular double Stone algebras, in turn, are definitionally equivalent to many other algebraic structures, and a comprehensive investigation of these connections can be found in [5]. In an ACA, the contact relation $\mathscr{C}$ splits into a *lower contact* $\mathscr{C}^i$ and an *upper contact* $\mathscr{C}^h$. In the present paper we fulfill the promise made in [2] and present relational logics for AAs and ACAs.

A general framework for designing proof systems for theories whose models involve relational structures is based on the methodology of relational proof systems in the style of [7] (RS systems). These consist of decomposition rules, specific rules and axiomatic sequences. A decomposition rule when applied to an expression of the theory returns a set of expressions which are syntactically simpler than the original one. Specific rules modify a sequence of formulas, and have the status of structural rules. These rules provide definitions of relational operators. In most instances, the rules are actually rule schemas. The role of axioms is played by axiomatic sequences. In an RS system, proving a theorem amounts to expanding a (proof) tree

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with the formula to be proved as the root, with the aim of closing all branches with axiomatic sequences. Every step of the expansion is based on either a decomposition rule or a specific rule. It should be mentioned that rules in RS systems go in both directions, that is, they preserve and reflect validity of expressions. A transfer of validity from the conclusion of a rule to the premise is needed for the soundness of the system, whereas the other direction is required for completeness. For a comprehensive exposition of RS system we invite the reader to consult [4], where, in particular, Chapter 25 describes the methodology.

For AAs and ACAs we have constructed sound and complete relational logics in two steps. First, relational versions of AA and ACA were obtained by expressing operators as relations with suitable arity and properties. In a second step, the corresponding relational logics were constructed.

We have implemented the AA and the ACA logic in Prolog. The number of rules is fairly large, hence, the search space is huge. Therefore, we have employed heuristics to reduce the search space to a manageable level.

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