Generalizations of Rough Set Tools inspired by Graph Theory

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During the years, several generalization of rough sets have been defined. They are based on a more general environment, such as a covering instead of a partition, a generic binary relation instead of a an equivalence one or a fuzzy setting instead of a Boolean. However, the main tools of the theory remain in the shadows. Notions such as the core, the reducts and the discernibility matrix are updated according to the new generalized environment but not directly considered as a possible way to generalize the theory.

On the contrary, in our studies on the connection between granular computing and graphs, we had the need to generalize some of these concepts. In the present work, we are going to study these new tools in the general case beyond the particular information tables arising from graphs.

As a first generalization, let us consider a simple undirected graph and interpret its adjacency matrix as an information table (so, attributes coincide with objects). Then, it often happens that the core of such an information table is empty [1], consequently, in this context, the notion of core becomes useless. In order to give a tool that is meaningful and useful, we thus introduced the idea of *extended core*.

Definition *We say that a subset* $C \subseteq Att$ *is* $\mathcal{I}$-essential *(or, more simply,* essential*) if :*

(i) $\pi_{Att \setminus C}(\mathcal{I}) \neq \pi_{Att}(\mathcal{I})$*;*

(ii) for all $D \subsetneqq C$*, we have that* $\pi_{Att \setminus D}(\mathcal{I}) = \pi_{Att}(\mathcal{I})$*.*

The extended core $ESS(\mathcal{I})$ *is the family of all the* $\mathcal{I}$*-essential subsets. If* $k \in \{1, \ldots, n\}$*, we set* $ESS_k(\mathcal{I}) := \{C \in ESS(\mathcal{I}) : |C| = k\}$ *and call it the* k-th core *of the information table* $\mathcal{I}$*.*

By definition, we have that $CORE(\mathcal{I}) = \bigcup\{C : C \in ESS_1(\mathcal{I})\}$. Hence, the subset family $ESS(\mathcal{I})$ is effectively an extension of the classical core of $\mathcal{I}$. Similarly, the notion of essential set extends the idea of *indispensable* attribute.

In the present work, we will see how to compute the extended core from the discernibility matrix and its relationship with the reducts. Basically, the reducts are exactly the minimal transversals of the hypergraph $(Att, ESS(\mathcal{I}))$. Otherwise stated, the conjunction of the elements of $ESS(\mathcal{I})$ is the standard discernibility function after the absorption law has been applied.

As a further generalization, in order to characterize a kind of symmetry in graphs, we introduced a generalized notion of discernibility function and discernibility matrix [2].

Definition *If $Z \subseteq U$ we define the* generalized discernibility function as

$$\Delta_{\mathcal{I}}(Z) := \{a \in Att \ : \ \exists \ z, z' \in Z : F(z,a) \neq F(z',a)\},$$

and also set

$$\Delta^c_{\mathcal{I}}(Z) := Att \setminus \Delta_{\mathcal{I}}(Z) = \{a \in Att \ : \ \forall \ z, z' \in Z, F(z,a) = F(z',a)\}$$

It is clear then that $\Delta^c_{\mathcal{I}}(Z)$ is the unique attribute subset C of $\mathcal{I}$ such that : $z \equiv_C z'$ for all $z, z' \in Z$ and such that if $A \subseteq Att$ and $z \equiv_A z'$ for all $z, z' \in Z$, then $A \subseteq C$.

If $Z = \{v_i, v_j\}$ then $\Delta_{\mathcal{I}}(Z)$ is equal to the entry $\delta_{\mathcal{I}}(v_i, v_j)$ of the (standard) discernibility matrix. In this sense, we can consider $\Delta_{\mathcal{I}} = \{\Delta_{\mathcal{I}}(Z) : Z \subseteq U\}$ a generalization of the discernibility matrix.

We notice that, if the information table is Boolean, then $\Delta^c(Z)$ resembles the *intension* of the set of objects Z in Formal Concept Analysis. In case of a many valued context, Δ^c could be considered a generalization of the FCA intension operator. This link with the FCA operator will be studied and the relationship of this discernibility matrix with reducts analyzed.

References

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