On Some Issues in the Foundation of Rough Sets

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In this presentation we shall raise the following issues connected with the discourse of rough set theory:
1. indistinguishability
2. definition of a rough set
3. language dependency, the problem of referent
4. vagueness
5. abstraction
6. application-friendliness

We shall argue that existing practices prevailing in rough set research fall short in addressing most of the issues.

After almost fifty years of rough-set studies it may be interesting, if not a necessity, to look into some basic aspects of its foundation.

Indistinguishability At the base level of all kinds of rough sets lies an indiscernibility relation (*ind*) which is at least reflexive and symmetric. This relation gives rise to the granulation $\mathcal{G}$ of the universe. That is, clustering of the objects: $\mathcal{G}_a$, the granule corresponding to the object a is $\mathcal{G}_a = \{x : a \; ind \; x\}$. This is true also for covering-based rough sets since from a covering $\mathcal{C}$ at first the granules viz., $N(a) = \cap\{C_i \in \mathcal{C} : a \in C_i\}$, $N_a = \cup\{C_i \in \mathcal{C} : a \in C_i\}$, and $P_a = \{x : \forall C_i(a \in C_i \Leftrightarrow x \in C_i)\}$ are formed.

One can consider elements belonging to the same granule as mutually indiscernible. While defining lower and upper approximations the granules are used differently. In fact any covering is itself a granulation. With the help of this, working granules $N(a)$, N_a, P_a etc. are formed. This indiscernibility is at the philosophical root of rough set theory. We talk about one individual but think about every one in the granule which it belongs to. This duality prevails althrough. Some objects are indistinguishable though not identical - like quantam objects (Logical and Philosophical Remarks on Quasi-Set Theory, Da Costa, Krause). Also this indistinguishability lies at the root of the Sorites type characteristic feature of vagueness, about which we shall discuss also.

With the help of the basic granules, approximations of a concept or set, a second-order entity is envisaged, and this is the main entry point to the application (or use) of granulation. A set 'A' is approximated by a pair $\underline{\mathcal{G}}(A)$ and $\overline{\mathcal{G}}(A)$ where usually $\underline{\mathcal{G}}(A) \subseteq A \subseteq \overline{\mathcal{G}}(A)$, but not always. Then a second layer of indiscernibility arises, that of the concepts. Two concepts A and B are indistinguishable if and only if they admit the same approximations with respect to a given granulation. At this level also a dichotomy prevails; concepts A and B are not the same, yet there is no means to distinguish them. Again, a name has different referents. In the conceptual framework of rough set theory this has to be reflected.

Definition There are several definitions of a rough set in the existing rough set literature. (1) A set 'A' is rough iff its boundary i.e., $\overline{\mathcal{G}}(A)$ - $\underline{\mathcal{G}}(A)$ is non-empty, in other words the boundary is 'thick'. According to this definition, an ordinary set is sometimes rough', and sometimes not. This definition is akin to saying a set is 'open' in the topological context. It is not that the ordinary sets are special cases of the concept 'rough sets'; 'rough' is an adjective to the word set.

On the other hand, considering 'rough set' as a single concept (akin to the term 'fuzzy set'), there are the following definitions.
(2) A rough set is a pair $\langle \underline{\mathcal{G}}(A), \overline{\mathcal{G}}(A) \rangle$ (or its equivalent).
(3) A rough set is an equivalence class $[A]_{\approx}$ with respect to the equivalence relation $\approx$ defined by $A \approx B$ iff $\underline{\mathcal{G}}(A) = \underline{\mathcal{G}}(B)$ and $\overline{\mathcal{G}}(A) = \overline{\mathcal{G}}(B)$.

According to this definition a pair $\langle \underline{\mathcal{G}}(A), \overline{\mathcal{G}}(A) \rangle$ where $\underline{\mathcal{G}}(A) = \overline{\mathcal{G}}(A) = A$ is also rough or equivalently $[A]_{\approx}$ is rough even though it is a singleton set containing only A. In this sense definitions (2) and (3) differ from (1). There is another definition in terms of definable sets. a set D is definable (with respect to $\mathcal{G}$) iff $\underline{\mathcal{G}}(D) = \overline{\mathcal{G}}(D) = D$.
(4) Any pair $\langle D_1, D_2 \rangle$ where D_1, D_2 are definable sets and $D_1 \subseteq D_2$ is called a rough set.

The advantage of this definition over (2) and (3) is that one needs not refer to the subsets A of the universe X while defining a rough set on X; only some kind of subsets of X are to be identified and pairs of this kind of subsets of X are rough sets. The basic motivation is expressed by Marek and Truczynski, "The emphasis on the set A present in the original definition of rough sets is what we strive here to free ourselves from." If A is already known why to approximate at all?

But none of the above definitions captures the above mentioned dichotomy. If $\langle \underline{A}, \overline{A} \rangle$ or $[A]_{\approx}$ or $\langle D_1, D_2 \rangle$ is a rough set then it is a unique entity, not that it is each of both A and B such that they have the same lower and upper approximatons. The rough set $\langle \phi, X \rangle$ where ϕ is the null set and X is the universe is not a single entity, it is the sign (name) denoting each set with ϕ as the lower approximation and X as the upper approximation. The name should have multiple referents - even one can think of variable referent.

There seems to be a hide and seek game between the description (language) and the objects described. All these descriptions miss this game.

Similarly, the other issues namely (3) to (6), in particular, the issue of vagueness will be discussed in some detail. The relationship between the criterion of vagueness by Shapiro and the existing definitions of rough sets will be highlighted.

References

1. M.K. Chakraborty, Mohua Banerjee, Rough sets: some foundational issues, Fundamenta Informaticae XX, 1-14, 2013.
2. M.K. Chakraborty, Membership function based rough set, International Journal of Approximate Reasoning 55, 402-411, 2014.
3. V.W. Marek, M. Truszczynski, Contributions to the theory of rough sets, Fundamenta Informaticae, 39(4), 389-409, 1999.
4. Z. Pawlak, Vagueness and uncertainty: a rough set perspective, Computational Intelligence: An International Journal, 11:217-232, 1995.
5. A. Skowron, Rough sets and vague concepts, Fundamenta Informaticae, 64:417-431, 2005.