APPROXIMATIONS IN INFORMATION NETS

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Abstract

We investigate infomorphisms between two information systems IS_1 and IS_2. Such infomorphisms make it possible to define semantics of some formulas over IS_2 by means of formulas over IS_1. The remaining formulas over IS_2 can be approximated by means of formulas over IS_1. The approximations are defined using rough set approach. We present definitions and examples of such approximations.

Keywords: rough set theory, classifications, infomorphisms, approximations, approximate reasoning, information nets

1 Introduction

In constructing of intelligent systems it is necessary to develop tools for approximate reasoning about vague and incomplete concepts in distributed environments [19], [13], [15], [16], [9], [10], [17].

In the paper we discuss relationships between information flow [2] and rough sets [8]. The former approach has been introduced to deal with reasoning in distributed systems while the latter makes possible to deal with incomplete and vague concepts.

Infomorphisms between information systems IS_1 and IS_2 are basic links between information systems. They make it possible to introduce logics of distributed systems modelled by nets (channels) of information systems connected by infomorphisms [2].

The aim of our project is to investigate if the logics defined by nets of information systems can be extended to the case of approximate reasoning. The paper realizes the first step in this direction. We distinguish formulas over IS_2 definable by means of formulas over IS_1 assuming there exists an infomorphism from IS_1 to IS_2. The formulas over IS_2 not-definable by formulas over IS_1 can be approximated by means of formulas over IS_1. The approximations are defined using the rough set approach. We present definitions and examples of such approximations. If there are infomorphisms from any information system from a given family to a given information system IS, then approximations of formulas (types [2]) over IS can be constructed by means of composition of patterns from IS definable in information systems from the family. Certainly, this requires the assumption that the set of formulas (types) of IS is closed with respect to the composition operation. The contribution of this article is the introduction of approximations of concepts not-definable exactly (or types, using terminology from [2]) in nets of information systems linked by infomorphisms. This makes it possible to reason in an approximate way in a given node of information net about concepts definable in the other nodes.

The paper is organized as follows. In Section 2 we show that classifications [2] and information systems [8] are equivalent. We also recall the infomorphism definition [2]. In Section 3 we discuss approximations of concepts in information systems linked by infomorphisms. An example of such approximations is also included. Approximations of concepts in information nets are introduced in Section 4 together with an illustrative example. Section 5 is dedicated to approximate reasoning in information nets.

2 Basic Concepts

In this section we present basic notions for our approach, i.e., information systems and infomorphisms.

2.1 Information Systems and Classifications

In this section we show that information systems [8] and classifications [2], [1], [6] are equivalent. First, let us recall these basic concepts.

Let $IS = (U, A)$ be an *information systems*, where U is a set of objects and A is a set of attributes. Let for every $a \in A$ V_a be a set of values of attribute a.

In [2] classifications are discussed. A *classification* is any tuple

$$\mathcal{A} = (\Sigma_{\mathcal{A}}, C, \models_{\mathcal{A}})$$

where $\Sigma_{\mathcal{A}}, C$ are sets called the set of *types* and the set of *tokens*, respectively and $\models_{\mathcal{A}}$ is a binary relation in $\Sigma_{\mathcal{A}} \times C$, i.e., $\models_{\mathcal{A}} \subseteq \Sigma_{\mathcal{A}} \times C$. The notation $x \models_{\mathcal{A}} \alpha$ reads "*x satisfies α relative to classification $\mathcal{A}$*" for $x \in C$ and $\alpha \in \Sigma_{\mathcal{A}}$ [3].

Any classification $\mathcal{A}$ defines an information system $IS_{\mathcal{A}} = (U, A)$ where $U = C$ and $A = \{a_\alpha\}_{\alpha \in \Sigma_{\mathcal{A}}}$ where $a_\alpha(x) = 1$ if and only if $x \models_{\mathcal{A}} \alpha$ for any type $\alpha \in \Sigma_{\mathcal{A}}$.

Any binary information system $IS = (U, A)$, i.e., an information system where $V_a = \{0, 1\}$ for any $a \in A$ defines a classification $\mathcal{A}_{IS} = (\Sigma_{\mathcal{A}}, C, \models_{\mathcal{A}})$ where $\Sigma_{\mathcal{A}} = \{\alpha_a\}_{a \in A}$, $C_{IS} = U$ and $x \models_{\mathcal{A}} \alpha_a$ if and only if $a(x) = 1$.

We have the following proposition:

Proposition 1 *For any binary information system IS and for any classification $\mathcal{A}$ we obtain*

1. $IS_{\mathcal{A}_{IS}} = IS$.
2. $\mathcal{A}_{IS_{\mathcal{A}}} = \mathcal{A}$.

For information system $IS = (U, A)$ with arbitrary attributes (i.e., multi-valued attributes) one can consider its binary representation, i.e., an information system $IS_{bin} = (U, A_{bin})$ where $A_{bin} = \{(a = v) : (a \in A \wedge v \in V_a) \wedge \exists c(c \in U \wedge a(c) = v)\}$ and $(a = v)(c) = 1$ if and only if $a(c) = v$. The indiscernibility relations [8] of IS and IS_{bin} are the same.

Hence, to this end we will not distinguish between information systems and classifications defined by them.

Let us consider one more classification related to information systems.

We denote by $\Sigma(IS)$ a set of formulas over IS. More precisely, the set $\Sigma(IS)$ is defined recursively by

1. $(a \in V) \in \Sigma(IS)$, for any $a \in A$ and $V \subseteq V_a$.
2. If $\alpha \in \Sigma(IS)$ then $\neg\alpha \in \Sigma(IS)$.
3. If $\alpha, \beta \in \Sigma(IS)$ then $\alpha \wedge \beta \in \Sigma(IS)$.
4. If $\alpha, \beta \in \Sigma(IS)$ then $\alpha \vee \beta \in \Sigma(IS)$.

The semantics of formulas from $\Sigma(IS)$ with respect to an information system IS is defined recursively by

1. $\|a \in V\|_{IS} = \{x \in U : a(x) \in V\}$.
2. $\|\neg\alpha\|_{IS} = U - \|\alpha\|_{IS}$.
3. $\|\alpha \wedge \beta\|_{IS} = \|\alpha\|_{IS} \cap \|\beta\|_{IS}$.
4. $\|\alpha \vee \beta\|_{IS} = \|\alpha\|_{IS} \cup \|\beta\|_{IS}$.

For all formulas $\alpha \in \Sigma(IS)$ and for all objects $x \in U$ we will denote $x \vDash_{IS} \alpha$ if and only if $x \in \|\alpha\|_{IS}$.

Now one can define a classification

$$\mathcal{B}ool_{IS} = (\Sigma(IS), U, \models_{IS})$$

where $c \models_{IS} \alpha$ if and only if $c \in \|\alpha\|_{IS}$.

Information systems (or classifications) are simple relational structures. However, they are enough general to represent information about features of complex objects perceived by different agents [5]. For example, structural objects can be represented by means of values of features related to their parts and relations between them. Hence, such relational structures are widely used in many areas including rough sets [8], [11] and information nets [2].

2.2 Infomorphisms

In this section we introduce the definition of infomorphism for two information systems. This notion is considered in [2], [6].

Definition 2 *If* $IS_1 = (U_1, A_1)$ *and* $IS_2 = (U_2, A_2)$ *are information systems and* $f : \Sigma(IS_1) \rightarrow \Sigma(IS_2)$, $g : U_2 \rightarrow U_1$, *then an infomorphism is a pair* (f, g) *of functions satisfying the following equivalence*

$$g(x) \models_{IS_1} \alpha \text{ if and only if } x \models_{IS_2} f(\alpha)$$

for all objects $x \in U_2$ *and for all formulas* $\alpha \in \Sigma(IS_1)$.

The infomorphism will be denoted shortly by $(f, g) : IS_1 \rightleftarrows IS_2$.

Proposition 3 *For any infomorphism* $(f, g) : IS_1 \rightleftarrows IS_2$ *we obtain the following equality*

$$g^{-1}(\|\alpha\|_{IS_1}) = \|f(\alpha)\|_{IS_2} \text{ for any } \alpha \in \Sigma(IS_1).$$

Proof. In fact $x \in g^{-1}(\|\alpha\|_{IS_1})$ if and only if $g(x) \in \|\alpha\|_{IS_1}$. The last condition is equivalent to $x \in \|f(\alpha)\|_{IS_2}$ which follows from the infomorphism definition.

Example 4 *General scheme of infomorphism is depicted in Figure 1, where*

- $IS_1 = (U_1, A_1)$ *and* $IS_2 = (U_2, A_2)$ *are information systems*
- U_1 *and* U_2 *are sets of objects.*
- A_1 *and* A_2 *are sets of attributes.*
- $\Sigma(IS_1)$ *and* $\Sigma(IS_2)$ *are sets of formulas over sets* A_1 *and* A_2 *of attributes, respectively.*
- f *is a function such that* $f : \Sigma(IS_1) \rightarrow \Sigma(IS_2)$
- $g : U_2 \rightarrow U_1$
- α *is a formula such that* $\alpha \in \Sigma(IS_1)$ *and* $f(\alpha) \in \Sigma(IS_2)$
- $\|\alpha\|_{IS_1} = \{x \in U_1 : x \models_{IS_1} \alpha\}$
- $\|f(\alpha)\|_{IS_2} = \{x \in U_2 : x \models_{IS_2} f(\alpha)\}$

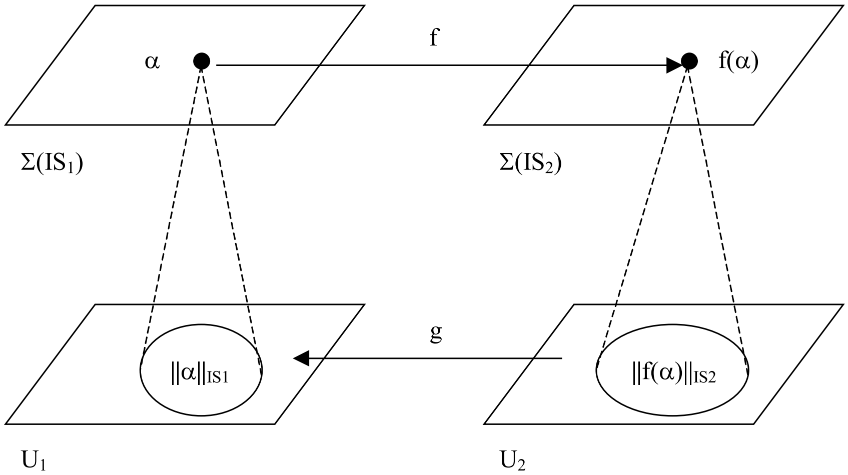

Figure 1: Infomorphism - General Scheme

3 Approximations and Infomorphisms

In this section, we discuss approximations of concepts definable by types in classifications. Assume $(f, g) : IS_1 \rightleftarrows IS_2$. We say that a type β from $\Sigma(IS_2)$ is $\Sigma(IS_1)$-*definable* if and only if there is $\alpha \in \Sigma(IS_1)$ such that $\beta = f(\alpha)$. Then by Proposition 3 we have $g^{-1}(\|\alpha\|_{IS_1}) = \|\beta\|_{IS_2}$ what explains why we say that β is $\Sigma(IS_1)$-definable by α. Hence, types of IS_2 from the f-image of $\Sigma(IS_1)$ are definable in IS_1.

Proposition 5 *If $(f, g) : IS_1 \rightleftarrows IS_2$ then any type from $f(\Sigma(IS_1))$ is $\Sigma(IS_1)$-definable.*

Now, we show that any $\alpha \in \Sigma(IS_2) - f(\Sigma(IS_1))$ can be approximated by types from $\Sigma(IS_2)$ (and $\Sigma(IS_1)$).

We define approximations analogously to the rough set approach [8]. For any type $\alpha \in \Sigma(IS_2) - f(\Sigma(IS_1))$ we define its $\Sigma(IS_1)$–lower and $\Sigma(IS_1)$–upper approximations by

$$\underline{\Sigma(IS_1)}\alpha = \bigcup_{\beta \in \Sigma(IS_1)} \{\|f(\beta)\|_{IS_2} : \|f(\beta)\|_{IS_2} \subseteq \|\alpha\|_{IS_2}\}$$

$$\overline{\Sigma(IS_1)}\alpha = \bigcup_{\beta \in \Sigma(IS_1)} \{\|f(\beta)\|_{IS_2} : \|f(\beta)\|_{IS_2} \cap \|\alpha\|_{IS_2} \neq \emptyset\}.$$

Proposition 6 *We have the following equalities:*

$$\underline{\Sigma(IS_1)}\alpha = \bigcup_{\beta \in \Sigma(IS_1)} \{g^{-1}\|\beta\|_{IS_1} : \|f(\beta)\|_{IS_2} \subseteq \|\alpha\|_{IS_2}\}$$

$$\overline{\Sigma(IS_1)}\alpha = \bigcup_{\beta \in \Sigma(IS_1)} \{g^{-1}\|\beta\|_{IS_1} : \|f(\beta)\|_{IS_2} \cap \|\alpha\|_{IS_2} \neq \emptyset\}.$$

Example 7 *Let $IS_1 = (U_1, A_1)$ and $IS = (U, A)$ be information systems, where $U_1 = \{x_1, \ldots, x_6\}$ and $U = U_1 \times \{y_1, y_2\}$ (see Table 1 and Table 2, respectively).*

Let $A_1 = \{a, b\}$, $V_a = \{v_1, v_2\}$ and $V_b = \{w_1, w_2\}$. We assume that $A = \{a', b', d\}$, $V_d = \{+, -\}$, $a'((x_i, y_j)) = a(x_i)$ and $b'((x_i, y_j)) = b(x_i)$, where $i = 1, \ldots, 6$ and $j = 1, 2$.

U_1/A_1	a	b
x_1	v_2	w_1
x_2	v_1	w_2
x_3	v_2	w_1
x_4	v_1	w_2
x_5	v_2	w_2
x_6	v_2	w_2

Table 1: Information System IS_1

Let $f : \Sigma(IS_1) \to \Sigma(IS)$ be such that for every formula $\alpha \in \Sigma(IS_1)$ $f(\alpha) = \alpha'$, where α' is obtained from α by translation of $a = v_i$ to $a' = v_i$ and $b = w_j$ to $b' = w_j$, where $i, j = 1, 2$.

U/A	a'	b'	d
(x_1, y_1)	v_2	w_1	$+$
(x_1, y_2)	v_2	w_1	$+$
(x_2, y_1)	v_1	w_2	$+$
(x_2, y_2)	v_1	w_2	$+$
(x_3, y_1)	v_2	w_1	$+$
(x_3, y_2)	v_2	w_1	$+$
(x_4, y_1)	v_1	w_2	$+$
(x_4, y_2)	v_1	w_2	$-$
(x_5, y_1)	v_2	w_2	$-$
(x_5, y_2)	v_2	w_2	$-$
(x_6, y_1)	v_2	w_2	$-$
(x_6, y_2)	v_2	w_2	$-$

Table 2: Information System IS

Let $g : U \to U_1$ be a projection, that is, for every $(x, y) \in U$ $g((x, y)) = x$.

Let α be a formula $d = +$. We have $\|\alpha\|_{IS_1} = X_+ = \{(x, y) \in U_2 : d((x, y)) = +\}$.

The IS_1-lower approximation of X_+ is equal to

$$\{(x_1, y_1), (x_1, y_2), (x_3, y_1), (x_3, y_2)\} = g^{-1}(\{x_1, x_3\}).$$

The IS_1-upper approximation of X_+ is equal to

$$\{(x_1, y_1), (x_1, y_2), (x_3, y_1), (x_3, y_2), (x_2, y_1), (x_2, y_2), (x_4, y_1), (x_4, y_2)\}$$
$$= g^{-1}(\{x_1, x_2, x_3, x_4\}).$$

One can observe that computed approximations coincide with the $\{a', b'\}$-lower and $\{a', b'\}$-upper approximations of the decision class X_+ using the rough set approach.

Assume $(f, g) : IS_1 \rightleftarrows IS_2$. We say that a type α from $\Sigma(IS_1)$ is $\Sigma(IS_2)$-*definable* if and only if $\|\alpha\|_{IS_1} \subseteq g(U_2)$. The latter assumed condition for α implies

$$g(\|f(\alpha)\|_{IS_2}) = \|\alpha\|_{IS_1}$$

what explains why we say that α is $\Sigma(IS_2)$-definable (by $f(\alpha)$).

Proposition 8 *If $(f, g) : IS_1 \rightleftarrows IS_2$ then any type $\alpha \in \Sigma(IS_1)$ satisfying $\|\alpha\|_{IS_1} \subseteq g(U_2)$ is $\Sigma(IS_2)$-definable (by $f(\alpha)$).*

4 Information Nets

Given two information systems IS_1 and IS_2, these information systems can be combined into a single information system IS. Thus we obtain information net with three information systems (see Figure 2).

In general, an *information net* [2] is a labelled graph with nodes labelled by information systems and edges labelled by infomorphisms between information systems of edges.

Let us recall that types (formulas) from $\Sigma(IS)$ are closed with respect to the conjunction ($\wedge$). Moreover, we assume $\|\alpha \wedge \beta\|_{IS} = \|\alpha\|_{IS} \cap \|\beta\|_{IS}$. In such case we can use the conjunction of patterns (types) from $f_1(\Sigma(IS_1))$ and $f_2(\Sigma(IS_2))$ for approximations of types (formulas) from $\Sigma(IS) - (f_1(\Sigma(IS_1)) \cup f_2(\Sigma(IS_2)))$.

We define approximations using an approach analogous to the rough set approach [8]. For type $\alpha \in (\Sigma(IS) - (f_1(\Sigma(IS_1)) \cup f_2(\Sigma(IS_2))))$ we define its $\Sigma(IS_1, IS_2)$–lower approximation by

$$\underline{\Sigma(IS_1, IS_2)}\alpha =$$
$$\bigcup_{\beta \in \Sigma(IS_1)} \bigcup_{\gamma \in \Sigma(IS_2)} \{\|f_1(\beta) \wedge f_2(\gamma)\|_{IS} : \|f_1(\beta) \wedge f_2(\gamma)\|_{IS} \subseteq \|\alpha\|_{IS}\}$$

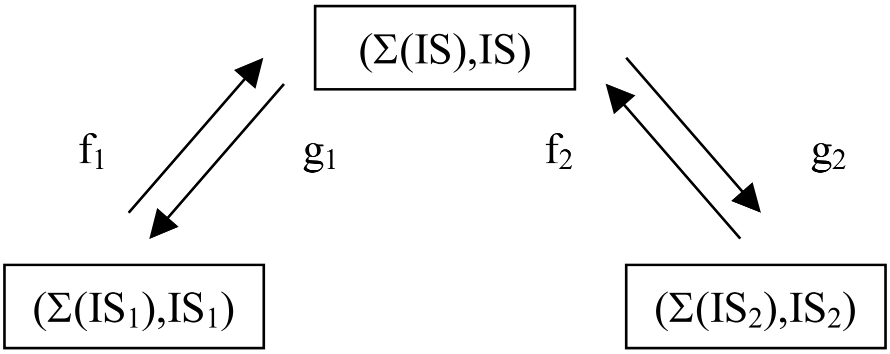

Figure 2: Information Net with Information Systems IS_1, IS_2, IS

and $\Sigma(IS_1, IS_2)$–upper approximation by

$$\overline{\Sigma(IS_1, IS_2)}\alpha =$$

$$\bigcup_{\beta\in\Sigma(IS_1)} \bigcup_{\gamma\in\Sigma(IS_2)} \{\|f_1(\beta) \wedge f_2(\gamma)\|_{IS} : \|f_1(\beta) \wedge f_2(\gamma)\|_{IS} \cap \|\alpha\|_{IS_2} \neq \emptyset\}.$$

Example 9 *Let us consider three information systems* $IS_1 = (U_1, A_1)$*,* $IS_2 = (U_2, A_2)$ *and* $IS = (U, A)$ *presented in Table 1, Table 3 and Table 4, respectively.*

U_2/A_2	c
y_1	1
y_2	0

Table 3: Information System IS_2

We obtain the following indiscernibility classes in U*:*

$$\{(x_1, y_1), (x_3, y_1)\}, \{(x_1, y_2), (x_3, y_2)\}, \{(x_2, y_1), (x_4, y_1)\},$$

$$\{(x_2, y_2), (x_4, y_2)\}, \{(x_5, y_1), (x_6, y_1)\}, \{(x_5, y_2), (x_6, y_2)\}.$$

We obtain the following $\Sigma(IS_1, IS_2)$*–approximations of the decision class*

$$X_+ = \{(x_1, y_1), (x_1, y_2), (x_2, y_1), (x_2, y_2), (x_3, y_1), (x_3, y_2), (x_4, y_1)\}.$$

The $\Sigma(IS_1, IS_2)$*–lower approximation:*

U/A	a'	b'	c'	d
(x_1, y_1)	v_2	w_1	1	$+$
(x_1, y_2)	v_2	w_1	0	$+$
(x_2, y_1)	v_1	w_2	1	$+$
(x_2, y_2)	v_1	w_2	0	$+$
(x_3, y_1)	v_2	w_1	1	$+$
(x_3, y_2)	v_2	w_1	0	$+$
(x_4, y_1)	v_1	w_2	1	$+$
(x_4, y_2)	v_1	w_2	0	$-$
(x_5, y_1)	v_2	w_2	1	$-$
(x_5, y_2)	v_2	w_2	0	$-$
(x_6, y_1)	v_2	w_2	1	$-$
(x_6, y_2)	v_2	w_2	0	$-$

Table 4: New Information System IS

$$\{(x_1, y_1), (x_3, y_1), (x_1, y_2), (x_3, y_2), (x_2, y_1), (x_4, y_1)\}.$$

The $\Sigma(IS_1, IS_2)$*–upper approximation:*

$$\{(x_1, y_1), (x_3, y_1), (x_1, y_2), (x_3, y_2), (x_2, y_1), (x_4, y_1), (x_2, y_2), (x_4, y_2)\}.$$

5 Approximate Reasoning in Information Nets

In this section, we present some brief remarks on approximate reasoning in information nets. We consider a problem related to reasoning about concepts defined (known) in some nodes (information systems) of an information net using concepts known in other nodes of this net. For example, one can ask if it is possible to solve the membership problem for concepts defined in a given node using some other nodes.

An approach to reasoning in information nets is presented in [2] where it is shown that some inference rules are preserving validity (or nonvalidity) of dependencies transformed from one node (information system) to another linked by an infomorphism [2].

In our paper, assuming $(f, g) : IS_1 \rightleftarrows IS_2$, we have shown using rough set approach that in case of a concept $X \subseteq U_2$ known at IS_2 but not definable at IS_1 it is still possible to reason in a given node IS_1 of information net about the concept X. In such case IS_1 can use approximations of the concept X to solve the membership problem. Let us explain this in more detail.

If $(f, g) : IS_1 \rightleftarrows IS_2$ then from the infomorphism definition we have

$$x \in \|f(\alpha)\|_{IS_2} \text{ if and only if } g(x) \in \|\alpha\|_{IS_1}$$

for any $x \in U_2$ and $\alpha \in \Sigma(IS_1)$.

Hence, for any $\beta \in f(\Sigma(IS_1))$ and $x \in U_2$ the membership query (at node IS_2): "*if* $x \in |\beta\|_{IS_2}$?" can be checked at node IS_1 by answering the membership query "*if* $g(x) \in \|\alpha\|_{IS_1}$?"

One can extend the above approach to concept approximations because they are constructed from definable sets. Hence, if at IS_1 it is possible to check that $g(x)$ for a given $x \in U_2$ belongs to the $\Sigma(IS_1)$-lower approximation of the concept X it means that such object x with certainty belongs to the concept X. Moreover, if $g(x)$ for some $x \in U_2$ does not belong to the $\Sigma(IS_1)$-upper approximation of the concept X it means that such object x with certainty belongs to the complement of the concept X.

Conclusions

This paper presents a first step towards approximate reasoning in nets of information systems. The approach is based on relationships between information flow and rough set approaches. In the presented approach information about whole objects in a given information system is linked by means of infomorphisms with information about their exact parts. We plan to extend an approach to the case of parts to a degree [13] and to use rough mereological approach in investigations of information nets. Such approach has close connections with information granule calculi in distributed systems.

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