49. Rough Measures and Integrals: A Brief Introduction

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This paper introduces a measure defined in the context of rough sets. Rough set theory provides a variety of set functions that can be studied relative to various measure spaces. In particular, the rough membership function is considered. The particular rough membership function given in this paper is a non-negative set function that is additive. It is an example of a rough measure. The idea of a rough integral is revisited in the context of the discrete Choquet integral that is defined relative to a rough measure. This rough integral computes a form of ordered, weighted "average" of the values of a measurable function. Rough integrals are useful in culling from a collection of active sensors those sensors with the greatest relevance in a problem-solving effort such as classification of a "perceived" phenomenon in the environment of an agent.

49.1 Introduction

This paper introduces a measure defined in the context of rough sets [49.3]. In this paper, we investigate measures defined on a family $\wp(X)$ of all subsets of a finite set X, i.e. on the powerset of X. A fundamental paradigm in rough set theory is set approximation. Hence, there is interest in discovering a family of measures useful in set approximation. By way of practical application, an approach to fusion of homogeneous sensors deemed relevant in a classification effort is considered (see, e.g., [49.6]). Application of rough integrals has also been considered recently relative to sensor signal classification by intelligent agents [49.8] and by web agents [49.9]. This research also has significance in the context of granular computing [49.10].

This paper is organized as follows. Section 49.2 presents a brief introduction to classical additive set functions. Basic concepts of rough set theory are presented in Section 49.3. The discrete Choquet integral is defined relative to a rough measure in Section 49.4. A brief introduction to sensor relevance is given in Section 49.5.

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49.2 Classical Additive Set Functions

This section gives a brief introduction to one form of additive set functions in measure theory. Let $card(X)$ denote the cardinality of a finite set X (i.e., the number of elements of set X).

Definition 49.2.1. Let X be a finite, non-empty set. A function $\lambda : \wp(X) \rightarrow \Re$ where $\Re$ is the set of all real numbers is called a *set function* on X.

Definition 49.2.2. Let X be a finite, non-empty set and let λ be a set function on X. The function λ is said to be *additive* on X iff $\lambda(A \cup B) = \lambda(A) + \lambda(B)$ for every $A, B \in \wp(X)$ such that $A \cap B = \emptyset$ (i.e., A and B are disjoint subsets of X).

Definition 49.2.3. Let X be a finite, non-empty set and let λ be a set function on X. A function λ is called to be *non-negative* on X iff $\lambda(Y) \geq 0$ for any $Y \in \wp(X)$.

Definition 49.2.4. Let X be a set and let λ be a set function on X. A function λ is called to be *monotonic* on X iff $A \subseteq B$ implies that $\lambda(A) \leq \lambda(B)$ for every $A, B \in \wp(X)$.

A brief introduction to the basic concepts in rough set theory (including the introduction of an additive rough measure) is briefly given in this section.

49.3 Basic Concepts of Rough Sets

Rough set theory offers a systematic approach to set approximation [49.2]. To begin, let $S = (U, A)$ be an information system where U is a non-empty, finite set of objects and A is a non-empty, finite set of attributes, where $a : U \rightarrow V_a$ for every $a \in A$. For each $B \subseteq A$, there is associated an equivalence relation $Ind_A(B)$ such that

$$Ind_A(B) = \{(x, x') \in U^2 \mid \forall a \in B.\mathrm{a(x) = a(x')}\}$$

If $(x, x') \in Ind_A(B)$, we say that objects x and x' are indiscernible from each other relative to attributes from B. The notation $[x]_B$ denotes equivalence classes of $Ind_A(B)$.

Definition 49.3.1. Let $S = (U, A)$ be an information system, $B \subseteq A$, $u \in U$ and let $[u]_B$ be an equivalence class of an object $u \in U$ of $Ind_A(B)$. The set function

$$\mu_u^B : \wp(U) \rightarrow [0, 1], \text{where } \mu_u^B(X) = \frac{card(X \cap [u]_B)}{card([u]_B)} \tag{49.1}$$

for any $X \in \wp(U)$ is called a *rough membership function (rmf)*.

The form of rough membership function in Def. 49.3.1 is slightly different from the classical definition where the argument of the rough membership function is an object x and the set X is fixed [49.3].

Definition 49.3.2. Let $u \in U$. A non-negative and additive set function $\rho_u : \wp(X) \to [0, \infty)$ defined by $\rho_u(Y) = \rho'(Y \cap [u]_B)$ for $Y \in \wp(X)$, where $\rho' : \wp(X) \to [0, \infty)$ is called a *rough measure* relative to $U/Ind_A(B)$ and u on the indiscernibility space $(X, \wp(X), U/Ind_A(B))$.

The rough membership function $\mu_u^B : \wp(X) \to [0, 1]$ is a non-negative set function [49.4].

Proposition 49.3.1. (Pawlak et al. [49.4]) The rough membership function μ_u^B as defined in Definition 49.3.1 (formula (49.1)) is additive on U.

Proposition 49.3.2. $(X, \wp(X), U/Ind_A(B), \{\mu_u^B\}_{u \in U})$ is a rough measure space over X and B.

Other rough measures based on upper {lower} approximations are possible but consideration of these other measures is outside the scope of this paper.

49.4 Rough Integrals

Rough integrals of discrete functions were introduced in [49.5]. In this section, we consider a variation of the Lebesgue integral, the discrete Choquet integral defined relative to a rough measure. In what follows, let $X = \{x_1, \ldots, x_n\}$ be a finite, non-empty set with n elements. The elements of X are indexed from 1 to n. The notation $X_{(i)}$ denotes the set $\{x_{(i)}, x_{(i+1)}, \ldots, x_{(n)}\}$ where $i \geq 1$ and $n = card(X)$. The subscript (i) is called a permutation index because the indices on elements of $X_{(i)}$ are chosen after a reordering of the elements of X. This reordering is "induced" by an external mechanism.

Definition 49.4.1. Let ρ be a rough measure on X where the elements of X are denoted by $x_1, \ldots, x_n$. The discrete Choquet integral of $f : X \to \Re^+$ with respect to the rough measure ρ is defined by

$$\int f \, d\rho = \sum_{i=1}^{n} (f(x_{(i)}) - f(x_{(i-1)}))\rho(X_{(i)})$$

where $\bullet_{(i)}$ specifies that indices have been permuted so that $0 \leq f(x_{(i)}) \leq \cdots \leq f(x_{(n)})$, $X_{(i)} := \{x_{(i)}, \ldots, x_{(n)}\}$, and $f(x_{(0)}) = 0$.

This definition of the Choquet integral is based on a formulation in Grabisch [49.1], and applied in [49.2], [49.7]. The rough measure $\rho(X_{(i)})$ value serves as a "weight" of a coalition (or combination) of objects in set $X_{(i)}$ relative to $f(x_{(i)})$. It should be observed that in general the Choquet integral has the effect of "averaging" the values of a measurable function. This averaging closely resembles the well-known Ordered Weighted Average (OWA) operator [49.11].

Proposition 49.4.1. Let $0 < s \leq r$. If $a(x) \in [s, r]$ for all $x \in X_a$, then $\int a \, d\mu_u^e \in (0, r]$ where $u \in U$.

49.5 Relevance of a Sensor

In this section, we briefly consider the measurement of the relevance of a sensor using a rough integral. A sensor is considered relevant in a classification effort in the case where $\int a \, d\mu_u^e$ for a sensor a is close enough to some threshold in a target interval of sensor values. Assume that a denotes a sensor that responds to stimuli with measurements that govern the actions of an agent. Let $\{a\} = B \subseteq A$ where $a : U \to [0, 0.5]$ where each sample sensor value $a(x)$ is rounded to two decimal places. Let $(Y, U - Y)$ be a partition defined by an expert and let $[u]_e$ denote a set in this partition containing u for a selected $u \in U$. We further assume the elements of $[u]_e$ are selected relative to an interval $(u - \varepsilon, u + \varepsilon)$ for a selected $\varepsilon \geq 0$. We assume a decision system (X_a, a, e) is given for any considered sensor a such that $X_a \subseteq U$, $a : X_a \to \Re^+$ and e is an expert decision restricted to X_a defining a partition $(Y \cap X_a, (U - Y) \cap X_a)$ of X_a. Moreover, we assume that $X_a \cap [u]_e \neq \emptyset$. The set $[u]_e$ is used to classify sensors and is given the name "classifier". Let $\bar{u}$ denote the average value in the classifier $[u]_e$, and let $\delta \in [0, 1]$. Then, for example, the selection R of the most relevant sensors in a set of sensors is found using

$$R = \left\{ a_i \in B \; : \; \left| \int a_i \, \mu_u^e - a(\bar{u}) \right| \leq \delta \right\}$$

In effect, the integral $\int a_i \, d\mu_u^e$ serves as a filter inasmuch as it "filters" out all sensors with integral values not close enough to $a(\bar{u})$.

49.6 Conclusion

Rough set theory provides a variety of set functions that can be studied relative to various measure spaces. In particular, the rough membership function is considered. The particular rough membership function given in this paper is a non-negative set function which is additive and, hence, is an example of a rough measure. We are interested in identifying those sensors considered relevant in a problem-solving effort. The rough integral introduced in this paper serves as a means of distinguishing relevant and non-relevant sensors in a classification effort.

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