Measures of Inclusion and Closeness of Information Granules: A Rough Set Approach

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Abstract. This article introduces an approach to measures of information granules based on rough set theory. The information granules considered in this paper are partially ordered multisets of sample sensor signal values, where it is possible for such granules to contain duplicates of the same values obtained in different moments of time. Such granules are also associated with a feature set in an information system. Information granules considered in this paper are collections of sample values derived from sensors that are modelled as continuous real-valued functions representing analog devices such as proximity (e.g., ultrasonic) sensors. The idea of sampling sensor signals is fundamental, since granule approximations and granule measures are defined relative to non-empty temporally ordered multisets of sample signal values. The contribution of this article is the introduction of measures of granule inclusion and closeness based on an indistinguishability relation that partitions real-valued universes into subintervals (equivalence classes). Such partitions are useful in measuring closeness and inclusion of granules containing sample signal values. The measures introduced in this article lead to the discovery of clusters of sample signal values.

Keywords: Closeness, inclusion, indistinguishability, information granule, measure, rough sets, sensor.

1 Introduction

This article introduces an approach to measures of a particular class of information granules based on rough set theory. Informally, a granule is a multiset (or bag) [19]-[20] of real-world objects that are somehow indistinguishable (e.g.,

J.J. Alpigini et al. (Eds.): RSCTC 2002, LNAI 2475, pp. 300–307, 2002.

water samples taken from the same source at approximately the same time), or similar (e.g., Chopin concerts), or which have the same functionality (e.g., unmanned helicopters). A multiset is a set where duplicates are counted. Examples of measures of granules are inclusion, closeness, size and enclosure. This paper is limited to a consideration of measures of inclusion based on a straightforward extension of classical rough membership functions [2], and the introduction of a measure of closeness of information granules. Measurement of sensor-based information granules have been motivated by recent studies of sensor signals [3], [5]-[6]. In this article, the term *sensor signal* is a non-empty, finite, discrete multiset of sample (either continuous or continuous) sensor signal values. Sample signal values are collected in temporally ordered multisets (repetitions of the same signal value are counted). In this article, classification of sensor signals is carried out using new forms of set approximation derived from classical rough set theory [1]-[2], [4]. This is made possible by introducing a number of additions to the basic building blocks of rough set theory, namely, (i) parameterized indistinguishability equivalence relation Ing defined relative to elements of an uncountable set, (ii) lower and upper approximation of information granules relative to a partition of an interval of the reals, and (iii) parameterized rough membership set function. A fundamental step in such a classification is a measure of the degree of overlap between a granule of sample sensor signal values and a target granule (collection ideal signal values for an application). Such a measure can be calibrated. A parameter δ in the definition of the relation Ing makes it possible to adjust the courseness or "granularity" of a partition of the subinterval of reals (universe) over which sensor signals are classified. Hence, such a measure has been used in the design of neurons in rough neural networks (see, e.g., [11]).

Granule approximation in this paper is cast in the context of infinite rather than finite universes. This study is motivated by the need to approximate and classify a number of different forms of uncountable sets (e.g., analog sensor signals such as speech, electrocardiograms, electroencephalograms). This is important in the context of parameterized approximation spaces used in designing intelligent systems [10], [12]-[15], [16]-[17], especially [12].

This paper is organized as follows. Section 2 presents introduces the parameterized indistinguishability relation and approximation of sets. This section also introduces a new form of rough membership set function. A natural extension of these ideas is the introduction of a rough measure space. Measurement of rough inclusion of granules is considered in Section 3.

2 Indistinguishability and Set Approximation

2.1 Indistinguishability Relation

To begin, let $S = (U, A)$ be an infinite information system where U is a non-empty set and A is a non-empty, finite set of attributes, where $a : U \to V_a$ and $V_a \subseteq \Re$ for every $a \in A$, so that $\Re \supseteq V = \bigcup_{a \in A} V_a$. Let $a(x) \geq 0$, $\delta > 0$ and let $\lfloor a(x)/\delta \rfloor$ denotes the greatest integer less than or equal to $a(x)/\delta$ ("floor" of a(x)/δ) whilst $\lceil a(x)/\delta \rceil$ denotes the least integer bigger than or equal to $a(x)/\delta$ ("ceiling" of $a(x)/\delta$) for attribute a. If $a(x) < 0$, then $\lfloor a(x)/\delta \rfloor = -\lceil |a(x)|/\delta \rceil$, where $|\bullet|$ denotes the absolute value of $\bullet$. The parameter δ serves as a means of computing a "neighborhood" size on real-valued intervals. Reals representing sensor measurements within the same subinterval bounded by $k\delta$ and $(k+1)\delta$ for integer k are considered δ-indistinguishable.

Definition 1. For each $B \subseteq A$, there is associated an equivalence relation $\text{Ing}_{A,\delta}(B)$ defined in (1).

$$\text{Ing}_{A,\delta}(B) = \left\{(x, x') \in \Re^2 \mid \forall a \in B.\, \lfloor a(x)/\delta \rfloor = \lfloor a(x')/\delta \rfloor \right\} \tag{1}$$

If $(x, x') \in \text{Ing}_{A,\delta}(B)$, we say that objects x and x' are indistinguishable from each other relative to attributes from B. A subscript Id denotes a set consisting of the identity sensor $id(x) = x$. The identity sensor id has been introduced to avoid the situation where there is more then one stimuli for which a sensor takes the same value (see example in the next section).

From (1), we can write $\text{Ing}_{A,\delta}(B)$ as in (2).

$$\text{Ing}_{A,\delta}(B \cup Id) = \left\{(x, x') \in \Re^2 \mid \lfloor \text{x}/\delta \rfloor = \lfloor \text{x}'/\delta \rfloor \wedge \forall a \in B.\, \lfloor a(x)/\delta \rfloor = \lfloor a(x')/\delta \rfloor \right\} \tag{2}$$

Proposition 1. $\text{Ing}_{A,\delta}(B \cup Id)$ is an equivalence relation.

The notation $[x]^{\delta}_{B \cup Id}$ denotes equivalence classes of $\text{Ing}_{A,\delta}$ $(B \cup Id)$. Further, partition $U/\ \text{Ing}_{A,\delta}(B \cup Id)$ denotes the family of all equivalence classes of relation $\text{Ing}_{A,\delta}$ $(B \cup Id)$ on U. For $X \subseteq U$, the set X can be approximated only from information contained in B by constructing a B-lower and a B-upper approximation denoted by $\underline{B}X$ and $\overline{B}X$, respectively. The B-lower approximation of X is the set $\underline{B}X = \left\{x \mid [x]^{\delta}_{B \cup Id} \subseteq X\right\}$ and the B-upper approximation of X is the set $\overline{B}X = \left\{x \mid [x]^{\delta}_{B \cup Id} \cap X \neq \emptyset\right\}$.

In cases where we need to reason about a sensor reading y instead of stimulus x, we introduce an equivalence class consisting of all points for which sensor readings are 'close' to y and define $[y]^{\delta}_{B \cup Id} = [x]^{\delta}_{B \cup Id}$ for x such that $a(x) = y$.

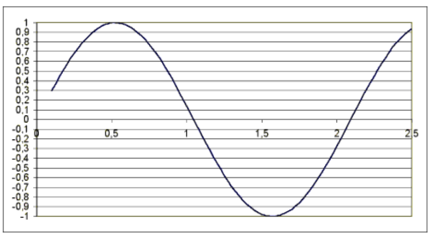

Fig. 1a. Sample Sensor Signal

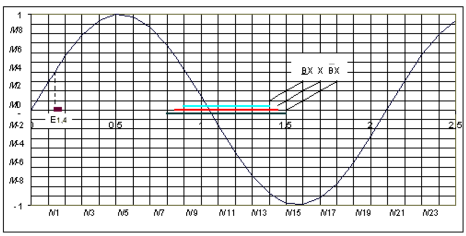

Fig. 1b. Sample approximations plus $E_{1,4}$ for $\delta = 0.1$

2.2 Sample Set Approximation

In this section, we want to consider a sample approximation of sensor stimuli relative to a set of points from a universe (subset of the reals). Consider the following continuous information system $S = (U, A)$, where U is a set of points in a subinterval [0, 2.5), and A is a set of sensors, e.g., $\{a \mid a_k(x)=\sin(kx)$ for $x \in [0, 2.5)$, k $\in Z^+$ (positive integers)$\}$.

Let parameter δ be set to 0.1 and let set $X = [0.85, 1.45]$. We want to consider $B \subset A$ such that $B = \{a_3(x)=\sin(3x)$ for $x \in [0, 2.5)\}$ (see Fig. 1a). A depiction of the upper and lower approximations of X is shown in Fig. 1b. To construct lower approximation, we need to find all δ-indistinguishability classes for δ=0.1 (sample δ-value). To elaborate using $id(x) = x$, the δ-indistinguishability relation $\mathrm{Ing}_{A,\delta}$ $(B \cup Id)$ in (2) can be instantiated with a specific set B and choice of δ. That is, consider $y' = a(x')$ for some $x' \in$ U (sample universe) and $\delta = 0.1$, and then consider

$$[y']^{0.1}_{\{a\}\cup Id} = \{x' \in U \mid \lfloor x'/0.1 \rfloor = \lfloor x/0.1 \rfloor \wedge \lfloor y'/0.1 \rfloor = \lfloor a(x')/0.1 \rfloor\}$$

Assume $\lfloor y'/0.1 \rfloor \in [n \cdot 0.1, (n+1) \cdot 0.1)$ and $\lfloor x/0.1 \rfloor \in [m \cdot 0.1, (m+1) \cdot 0.1)$, for some m, n, and obtain (3).

$$[y']^{0.1}_{\{a\}\cup Id} = \{x' \in U \mid \lfloor x'/0.1 \rfloor = m \wedge \lfloor y'/0.1 \rfloor = n\} \tag{3}$$

For simplicity, let N_n denote the interval $[n \cdot 0.1, (n+1) \cdot 0.1)$, let M_m denote the interval $[m \cdot 0.1, (m+1) \cdot 0.1)$. In addition, let $E_{n,m}$ denote the equivalence class given in (3), where n, m denote integers. For $y = 0.45$ $a(0.155) = \sin(0.466) = 0.45$ $(0.45 \in N_4, 0.155 \in M_1)$ so that $0.155 \in E_{1,4}$ (see Fig. 2). But also $a(0.891) = 0.45$ and $a(2.249) = 0.45$, so $0.891 \in E_{8,4}$ and $2.249 \in E_{22,4}$. For example $E_{1,4} = [0.137, 0.174)$, $E_{1,5} = [0.174, 0.2)$ and $E_{1,6} = \emptyset$ (empty set). $\underline{B}X = \bigcup_{n=0}^{24} \bigcup_{m=-1}^{10} E_{n,m}$ where $E_{n,m} \subseteq X$. Then the 0.1-lower approximation of X is $\underline{B}X = [0.9, 1.4)$. Similarly, we can find upper approximation of X to be $\overline{B}X = [0.8, 1.5)$. In the following sections, we write $[y]^{\delta}_{B}$ instead of $[y]^{\delta}_{B \cup Id}$ because we consider only sensor values relative to partitions of the universe using $\mathrm{Ing}_{A,\delta}$ (B).

2.3 Rough Membership Set Function

In this section, a set function form of the traditional rough membership function is presented

Definition 2. Let $S = (U, A)$ be an information system with non-empty set U and non-empty set of attributes A. Further, let $B \subseteq A$ and let $[y]^{\delta}_{B}$ be an equivalence class of any sensor reading $y \in \Re$. Let ρ be a measure of a set $X \in \wp(U)$, where $\wp(U)$ is a class (set of all subsets of U). Then for any $X \in \wp$(U) the *rough membership set function (rmf)* $\mu^{B,\delta}_{y} : \wp(U) \to [0, 1]$ is defined in (4).

$$\mu^{B,\delta}_{y}(X) = \frac{\rho\left(X \cap [y]^{\delta}_{B}\right)}{\rho\left([y]^{\delta}_{B}\right)} \tag{4}$$

If $\rho([y]^{\delta}_{B})$=0, then of course $\rho(X \cap [y]^{\delta}_{B}) = 0$ and in this situation we consider symbol $\frac{0}{0}$ to be equal to 0.

A form of rough membership set function for non-empty, finite sets was introduced in [3]. Definition 2 is slightly different from the classical definition where the argument of the rough membership function is an object x and the set X is fixed [2].

2.4 Rough Measures

In what follows, a distance metric is introduced that will make it possible to measure the closeness of information granules. Recall that a function d: $X \times X \to \Re$ is called a metric on a set X if and only if for any x, y, $z \in X$ the function d satisfies the following three conditions: (i) $d(x, y) = 0$ if and only if $x = y$; (ii) $d(x, y) = d(x, y)$; (iii) $d(x, y) + d(y, z) \geqslant d(x, z)$.

Proposition 2. Let $S = (U, A)$ be an information system and let $\rho(Y)$ be defined as $\int\limits_{x \subseteq Y} 1\, dx$.The function $\mu^{\delta}_{B,y} : \wp(U) \to \Re$ in (5) is measure of a set $X \subseteq U$.

$$\mu^{\delta}_{B,y}(X) = \sum_{[y']^{\delta}_{B} \subseteq \overline{B}X} \frac{\rho(X \cap [y']^{\delta}_{B})}{(d([y']^{\delta}_{B}, [y]^{\delta}_{B}) + 1) \cdot \rho([y']^{\delta}_{B})} \tag{5}$$

where $d(\bullet)$ denotes a metric on the partition $U/\mathrm{Ing}_{A,\delta}(B)$ of U defined by equivalence relation $\mathrm{Ing}_{A,\delta}(B)$. The formula (5) may be written as shown in (6)

$$\mu^{\delta}_{B,y}(X) = \frac{\rho(X \cap [y]^{\delta}_{B})}{\rho([y]^{\delta}_{B})} + \sum_{\left\{\begin{array}{l}[y']^{\delta}_{B} \subseteq \overline{B}X \\ [y']^{\delta}_{B} \neq [y]^{\delta}_{B}\end{array}\right.} \frac{\rho(X \cap [y']^{\delta}_{B})}{\rho([y']^{\delta}_{B})} \cdot \frac{1}{d([y']^{\delta}_{B}, [y]^{\delta}_{B}) + 1} \tag{6}$$

It is clearly seen that $\mu^{\delta}_{B,y}$ defined in (6) is measure (4) completed with a sum of analogous measures for the remaining equivalence classes weighted by the reverse of distance (plus one) between distinct class $[y]^{\delta}_{B}$ and the remaining equivalence classes. The ratio $1/d([y']^{\delta}_{B}, [y]^{\delta}_{B}) + 1$ serves as a weight of the sum in (6). Thanks to the number 1 in the denominator of this weight, it is possible to include measure (4) as a term in the sum in (6). To obtain values in the interval [0, 1] for the measure (5), the normalization coefficient α(y) in (7) is introduced.

$$\alpha(y) = \frac{1}{\sum\limits_{[y']^{\delta}_{B} \subseteq \overline{B}X} \frac{1}{d([y']^{\delta}_{B}, [y]^{\delta}_{B}) + 1}} \tag{7}$$

As a result, the following proposition holds.

Proposition 3. Let $S = (U, A)$ be an information system and let $\rho(Y)$ be defined as $\int\limits_{x \subseteq Y} 1dx$. If d is a metric defined on the set $U/\mathrm{Ing}_{A,\delta}(B)$ and $\alpha(y)$ is as in formula (7), then function $\mu^{\delta}_{B,y} : \wp(U) \to [0, 1]$ such that

$$\mu_{B,y}^{\delta}(X) = \alpha(y) \cdot \sum_{[y']_B^{\delta} \subseteq \overline{B}X} \frac{\rho(X \cap [y']_B^{\delta})}{(d([y']_B^{\delta}, [y]_B^{\delta}) + 1) \cdot \rho([y']_B^{\delta})} \tag{8}$$

is a measure on the set $\wp(U)$.

Consider the sample universe U = [0, 0.4)×[0, 0.3), a finite sample $X \subset U$ as shown in Fig. 2. The equivalence relation $\text{Ing}_{A,\delta}(B)$ partitions this sample universe as it is shown on Figure 2. Let us assume that each equivalence class consists of just 8 points. Every equivalence class (also called a mesh cell) is numbered by a pair of indices $E_{n,m}$. Notice that two sensor values are approximately equal in $E_{0,0}$, and are not considered duplicates. In addition, sensor values in $E_{0,0}$ are time-ordered with the relation $\leqslant_{\text{before}}$. For example, assume that a(x(t)) occurs before a(x(t')). Then we write a(x(t)) $\leqslant_{\text{before}}$ a(x(t')). In effect, $E_{0,0}$ and every other cell in the mesh in Fig. 2 constitutes a temporally ordered multiset. For such an information system and its partition $U/\text{Ing}_{A,\delta}(B)$, a well-known maximum metric is chosen, $d(E_{n1,m1}, E_{n2,m2}) = \max\{\ |n_1 - n_2|\ ,\ |m_1 - m_2|\ \}$. Finding equivalence class with the biggest measure of Xin sense of (5) leads to choice of $E_{0,0}$ while applying measure (8) gives as result class $E_{3,1}$. If we are interested in finding single equivalence class with the bigger (or least) degree of overlapping with set X, then measure (4) should be chosen, but when we want to find a group of 'neighbour' (in sense of "close") equivalence classes that overlap with X in the biggest (least) degree, then measure (8) is suggested. Figure 2

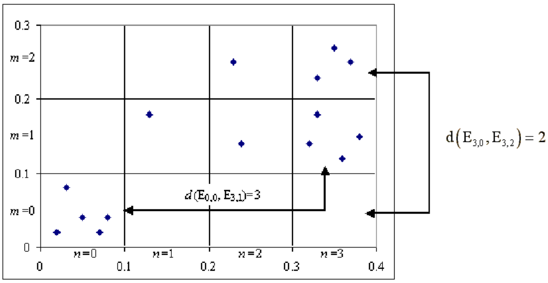

Fig. 2. Sample Distance Measurements in a δ-mesh

shows how the maximum metric measures distance. For example, $d(E_{3,0},E_{3,2})$ = max{ | 3-3 | , | 2-0 | }=2 between equivalence classes $E_{3,0}$ and $E_{3,2}$ may be of interest in cases where measurement of the separation between clusters (i.e., multiset that is the union of sensor values in a mesh cell and in neighboring cells) of sample sensor values is important (e.g., separation of cells in a mesh covering a control system performance map that contains "islands" of system response values, some normal and some verging on chaotic behavior as in [21], [22]).

3 Conclusion

Measures of inclusion and closeness of information granules have been introduced in the context of rough set theory. The partition of a universe using Ing results in a mesh of cells (called a δ-mesh), where each cell of the mesh represents an equivalence class. The configuration of cells in a δ-mesh yields a useful granule measure. That is, a measure of closeness of a pair of information granules contained in cells of the δ-mesh results from determining the number of cells separating members of the pair using a distance metric. Using a combination of the distance metric and a form of thresholding on the search space in a δ-mesh, the first of a family of algorithms for finding clusters of sensor values has been introduced. For simplicity, this algorithm has been restricted to δ-meshes covering a finite number of sample values for a single sensor. In future work, this algorithm will be extended to find clusters of sample values for more than one sensor. In addition, a calibration algorithm for finding an appropriate δ used to construct a δ-mesh will be introduced in further work on the problem of discovering clusters (granules) of sample sensor values.

Acknowledgements. The research of James Peters has been supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) research grant 185986 and grants from Manitoba Hydro, and University of Information Technology and Management, Rzeszów, Poland. The research of Maciej Borkowski has been supported by a grant from Manitoba Hydro. The research of Andrzej Skowron and Zbigniew Suraj has been supported by grant 8 T11C 025 19 from the State Committee for Scientific Research (KBN). Moreover, the research of Andrzej Skowron has been supported by a grant from the Wallenberg Foundation.

References

1. Z. Pawlak, *Rough Sets: Theoretical Aspects of Reasoning About Data*, Boston, MA, Kluwer Academic Publishers, 1991.
2. Z. Pawlak, A. Skowron, Rough membership functions. In: R. Yager, M. Fedrizzi, J. Kacprzyk (Eds.), *Advances in the Dempster-Shafer Theory of Evidence*, NY, John Wiley & Sons, 1994, 251–271.
3. Z. Pawlak, J.F. Peters, A. Skowron, Z. Suraj, S. Ramanna, M. Borkowski, Rough measures: Theory and Applications. In: S. Hirano, M. Inuiguchi, S. Tsumoto (Eds.), Bulletin of the International Rough Set Society, vol. 5, no. 1 / 2, 2001, 177–184.
4. Z. Pawlak, On rough derivatives, rough integrals, and rough differential equations. ICS Research Report 41/95, Institute of Computer Science, Nowowiejska 15/19, 00-665 Warsaw, Poland, 1995.
5. J.F. Peters, V. Degtyaryov, M. Borkowski, S. Ramanna, Line-crawling robot navigation: Rough neurocomputing approach. In: C. Zhou, D. Maravall, D. Ruan, Fusion of Soft Computing and Hard Computing for Autonomous Robotic Systems. Berlin: Physica-Verlag, 2002 [to appear].

6. J.F. Peters, S. Ramanna, A. Skowron, J. Stepaniuk, Z. Suraj, M. Borkowski, Sensor fusion: A rough granular approach. In: *Proc. Joint 9th International Fuzzy Systems Association (IFSA) World Congress and 20th North American Fuzzy Information Processing Society (NAFIPS) Int. Conf.*, Vancouver, British Columbia, Canada, 25-28 June 2001, 1367–1372.
7. J.F. Peters, S. Ramanna, L. Han, The Choquet integral in a rough software cost estimation system. In: M. Grabisch, T. Murofushi, M. Sugeno (Eds.), *Fuzzy Measures and Integrals: Theory and Applications.* (Springer-Verlag, Heidelberg, Germany, 2000) 392–414.
8. J.F. Peters, S. Ramanna, M. Borkowski, A. Skowron: Approximate sensor fusion in a navigation agent, in: N. Zhong, J. Liu, S. Ohsuga and J. Bradshaw (Eds.), Intelligent agent technology: Research and development. Singapore: World Scientific Publishing, 2001, 500–504.
9. J.F. Peters, S. Ramanna, A. Skowron, M. Borkowski: Wireless agent guidance of remote mobile robots: Rough integral approach to sensor signal analysis. In: N. Zhong, Y.Y. Yao, J. Liu, S. Ohsuga (Eds.), Web Intelligence, Lecture Notes in Artificial Intelligence 2198. Berlin: Springer-Verlag, 2001, 413–422.
10. L. Polkowski, A. Skowron, Towards adaptive calculus of granules. In: Proc. of the Sixth Int. Conf. on Fuzzy Systems (FUZZ-IEEE'98), Anchorage, Alaska, 4-9 May 1998, 111–116.
11. S.K. Pal, L. Polkowski, A. Skowron, Rough-neuro computing: Technologies in Computing with Words. Berlin: Physica-Verlag, 2002 [to appear].
12. A. Skowron, Toward intelligent systems: Calculi of information granules. In: S. Hirano, M. Inuiguchi, S. Tsumoto (Eds.), Bulletin of the International Rough Set Society, vol. 5, no. 1 / 2, 2001, 9–30.
13. A. Skowron, J. Stepaniuk, S. Tsumoto, Information granules for spatial reasoning, Bulletin of the International Rough Set Society, vol. 3, no. 4, 1999, 147–154.
14. A. Skowron, J. Stepaniuk, Constructive information granules. In: Proc. of the 15th IMACS World Congress on Scientific Computation, Modelling and Applied Mathematics, Berlin, Germany, 24-29 August 1997. Artificial Intelligence and Computer Science 4, 1997, 625-630.
15. A. Skowron, J. Stepaniuk, J.F. Peters, Extracting patterns using information granules. In: S. Hirano, M. Inuiguchi, S. Tsumoto (Eds.), Bulletin of the International Rough Set Society, vol. 5, no. 1 / 2, 2001, 135–142.
16. A. Skowron, J. Stepaniuk, Information granules and approximation spaces. In: Proc. of the 7th Int. Conf. on Information Processing and Management of Uncertainty in Knowledge-based Systems, Paris, France, 6-10 July 1998, 1354–1361.
17. A. Skowron, J. Stepaniuk, J.F. Peters, Hierarchy of information granules. In: H.D. Burkhard, L. Czaja, H.S. Nguyen, P. Starke (Eds.), *Proc. of the Workshop on Concurrency, Specification and Programming*, Oct. 2001,Warsaw,Poland, 254–268.
18. P.R. Halmos, Measure Theory. London: D. Van Nostrand Co., Inc., 1950.
19. R. R. Yager, On the theory of bags, Intern. J. General Systems, 13 (1986), 23–37.
20. A. Syropoulos, Mathematics of multisets, in Pre-proceedings of the Workshop on Multiset Processing, Curtea de Arges, Romania, 2000 (C.S. Calude, M.J. Dinneen, Gh. Paun, eds.), CDMTCS Res. Report 140, Auckland Univ., 2000, 286–295.
21. J.J. Alpigini, J.F. Peters, Dynamic visualization with rough performance maps. In: W.Ziarko, Y.Yao (Eds.), RSCTC'00, Lectures Notes in Artificial Intelligence 2005. Berlin: Springer-Verlag, 2001, 90–97.
22. J.J. Alpigini, Measures of closeness of performance map information granules: A rough set approach. In: RSCTC'02 [submitted].