Approximation of Information Granule Sets

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Abstract. The aim of the paper is to present basic notions related to granular computing, namely the information granule syntax and semantics as well as the inclusion and closeness (similarity) relations of granules. In particular, we discuss how to define approximation of complex granule sets using the above notions.

1 Introduction

We would like to discuss briefly an example showing a motivation for our work [6]. Let us consider a team of agents recognizing the situation on the road. The aim is to classify a given situation as, e.g., dangerous or *not*. This soft specification granule is represented by a family of information granules called case *soft patterns* representing cases, like cars are *too close*. The whole scene (actual situation on the road) is decomposed into regions perceived by local agents. Higher level agents can reason about regions observed by team of their children agents. They can express in their own languages features used by their children. Moreover, they can use new features like attributes describing relations between regions perceived by children agents. The problem is how to organize agents into a team (having, e.g., tree structure) with the property that the information granules synthesized by the team from input granules (being perceptions of local agents from sensor measurements) will identify the situation on the road in the following sense: the granule constructed by the team from input granules representing the situation on the road is sufficiently close to the soft specification granule named dangerous if and only if the situation on the road is really dangerous. We expect that if the team is returning a granule sufficiently close to the soft specification granule dangerous then also a special case of the soft pattern dangerous is identified helping to explain the situation.

The aim of our project is to develop foundations for this kind of reasoning. In particular it is necessary to give precise meaning to the notions like: information granules, soft information granules, closeness of information granules in

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satisfactory degree, information granules synthesized by team of agents etc. The presented paper realizes the first step toward this goal.

The general scheme is depicted in Figure 1.

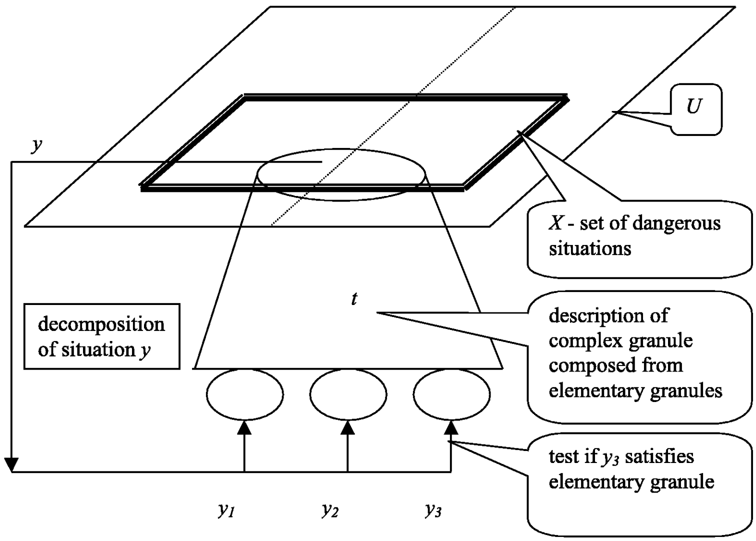

Fig. 1. Illustrative Example

To sum up, we consider a set of agents Ag. Each agent is equipped with some approximation spaces (defined using rough set approach [1]). Agents are cooperating to solve a problem specified by a special agent called *customer-agent.* The result of cooperation is a scheme of agents. In the simplest case the scheme can be represented by a tree labeled by agents. In this tree leaves are delivering some information granules (representing of perception in a given situation by leaf agents) and any non-leaf agent $ag \in Ag$ is performing an operation $o(ag)$ on approximations of granules delivered by its children. The root agent returns an information granule being the result of computation by the scheme on granules delivered by leaf agents. It is important to note that different agents use different languages. Thus granules delivered by children agents to their father can be usually perceived by him in an approximate sense before he can perform any operation on delivered granules.

In particular, we point out in the paper a problem of approximation of information granule sets and we show that the first step toward such a notion is similar to the classical rough set approach.

2 Syntax and Semantics of Information Granules

In this section we will consider several examples of information granule constructions. We present now the syntax and semantics of information granules. In the following section we discuss the inclusion and closeness relations for granules.

Elementary granules. In an information system $IS = (U, A)$, elementary granules are defined by $EF_B(x)$, where EF_B is a conjunction of selectors of the form $a = a(x)$, $B \subseteq A$ and $x \in U$. For example, the meaning of an elementary granule $a = 1 \wedge b = 1$ is defined by

$$\|a = 1 \wedge b = 1\|_{IS} = \{x \in U : a(x) = 1 \;\&\; b(x) = 1\}.$$

Sequences of granules. Let us assume that S is a sequence of granules and the semantics $\|\bullet\|_{IS}$ in IS of its elements have been defined. We extend $\|\bullet\|_{IS}$ on S by $\|S\|_{IS} = \{\|g\|_{IS}\}_{g \in S}$.

Example 1. Granules defined by rules in information systems are examples of sequences of granules. Let IS be an information system and let (α, β) be a new information granule received from the rule **if** α **then** β where α, β are elementary granules of IS. The semantics $\|(\alpha, \beta)\|_{IS}$ of (α, β) is the pair of sets $(\|\alpha\|_{IS}, \|\beta\|_{IS})$.

Sets of granules. Let us assume that a set G of granules and the semantics $\|\bullet\|_{IS}$ in IS for granules from G have been defined. We extend $\|\bullet\|_{IS}$ on the family of sets $H \subseteq G$ by $\|H\|_{IS} = \{\|g\|_{IS} : g \in H\}$.

Example 2. One can consider granules defined by sets of rules. Assume that there is a set of rules $Rule\_Set = \{(\alpha_i, \beta_i) : i = 1, \ldots, k\}$. The semantics of $Rule\_Set$ is defined by

$$\|Rule\_Set\|_{IS} = \{\|(\alpha_i, \beta_i)\|_{IS} : i = 1, \ldots, k\}.$$

Example 3. One can also consider as set of granules a family of all granules $(\alpha, Rule\_Set(DT_\alpha))$, where α belongs to a given subset of elementary granules.

Example 4. Granules defined by sets of decision rules corresponding to a given evidence are also examples of sequences of granules. Let $DT = (U, A \cup \{d\})$ be a decision table and let α be an elementary granule of $IS = (U, A)$ such that $\|\alpha\|_{IS} \neq \emptyset$. Let $Rule\_Set(DT_\alpha)$ be the set of decision rules (e.g. in minimal form) of the decision table $DT_\alpha = (\|\alpha\|_{IS}, A \cup \{d\})$ being the restriction of DT to objects satisfying α. We obtain a new granule $(\alpha, Rule\_Set(DT_\alpha))$ with the semantics

$$\|(\alpha, Rule\_Set(DT_\alpha))\|_{DT} = (\|\alpha\|_{IS}, \|Rule\_Set(DT_\alpha)\|_{DT}).$$

This granule describes a decision algorithm applied in the situation characterized by α.

Extension of granules defined by tolerance relation. We present examples of granules obtained by application of a tolerance relation.

Example 5. One can consider extension of elementary granules defined by tolerance relation. Let $IS = (U, A)$ be an information system and let τ be a tolerance relation on elementary granules of IS. Any pair (α, τ) is called a *τ-elementary granule.* The semantics $\|(\alpha,\tau)\|_{IS}$ of (α, τ) is the family $\{\|\beta\|_{IS} : (\beta,\alpha) \in \tau\}$.

Example 6. Let us consider granules defined by rules of tolerance information systems. Let $IS = (U, A)$ be an information system and let τ be a tolerance relation on elementary granules of IS. If **if** α **then** β is a rule in IS then the semantics of a new information granule $(\tau : \alpha, \beta)$ is defined by $\|(\tau : \alpha,\beta)\|_{IS} = \|(\alpha,\tau)\|_{IS} \times \|(\beta,\tau)\|_{IS}$.

Example 7. We consider granules defined by sets of decision rules corresponding to a given evidence in tolerance decision tables. Let $DT = (U, A \cup \{d\})$ be a decision table and let τ be a tolerance on elementary granules of $IS = (U, A)$. Now, any granule $(\alpha, Rule\_Set\,(DT_\alpha))$ can be considered as a representative of information granule cluster $(\tau : (\alpha, Rule\_Set\,(DT_\alpha)))$ with the semantics

$$\|(\tau : (\alpha, Rule\_Set\,(DT_\alpha)))\|_{DT} = \left\{\|(\beta, Rule\_Set\,(DT_\beta))\|_{DT} : (\beta,\alpha) \in \tau\right\}.$$

Labeled graph granules. We discuss graph granules and labeled graph granules as notions extending previously introduced granules defined by tolerance relation.

Example 8. Let us consider granules defined by pairs (G, E), where G is a finite set of granules and $E \subseteq G \times G$. Let $IS = (U, A)$ be an information system. The semantics of a new information granule (G, E) is defined by $\|(G,E)\|_{IS} = (\|G\|_{IS}, \|E\|_{IS})$, where $\|G\|_{IS} = \{\|g\|_{IS} : g \in G\}$ and $\|E\|_{IS} = \{(\|g\|, \|g'\|) : (g, g') \in E\}$.

Example 9. Let G be a set of granules. Labeled graph granules over G are defined by (X, E, f, h), where $f : X \to G$ and $h : E \to P(G \times G)$. We also assume one additional condition
if $(x, y) \in E$ then $(f(x), f(y)) \in h(x, y)$.

The semantics of labeled graph granule (X, E, f, h) is defined by

$$\{(\|f(x)\|_{IS}, \|h(x,y)\|_{IS}, \|f(y)\|_{IS}) : (x,y) \in E\}.$$

Let us summarize the above presented considerations. One can define the set of granules G as the least set containing a given set of elementary granules G_0 and closed with respect to the defined above operations of new granule construction.

We have the following examples of granule construction rules:

$$\frac{\alpha_1, \ldots, \alpha_k\text{- elementary granules}}{\{\alpha_1, \ldots, \alpha_k\}\text{- granule}}$$

$$\frac{\alpha_1, \alpha_2\text{- elementary granules}}{(\alpha_1, \alpha_2)\text{- granule}}$$

$$\frac{\alpha\text{- elementary granule },\tau\text{- tolerance relation on elementary granules}}{(\tau:\alpha)\text{- granule}}$$

$$\frac{G\text{- a finite set of granules },E\subseteq G\times G}{(G,E)\text{- granule}}$$

Let us observe that in case of granules constructed with application of tolerance relation we have the rule restricted to elementary granules. To obtain a more general rule like

$$\frac{\alpha\text{- graph granule },\tau\text{- tolerance relation on graph granules}}{(\tau:\alpha)\text{- granule}}$$

it is necessary to extend the tolerance (similarity, closeness) relation on more complex objects. We discuss the problem of closeness extension in the following section.

3 Granule Inclusion and Closeness

In this section we will discuss inclusion and closeness of different information granules introduced in the previous section. Let us mention that the choice of inclusion or closeness definition depends very much on the area of application and data analyzed. This is the reason that we have decided to introduce a separate section with this more subjective part of granule semantics.

The inclusion relation between granules G, G' of degree at least p will be denoted by $\nu_p(G, G')$. Similarly, the closeness relation between granules G, G' of degree at least p will be denoted by $cl_p(G, G')$. By p we denote a vector of parameters (e.g. positive real numbers).

A general scheme for construction of hierarchical granules and their closeness can be described by the following recursive meta-rule: if granules of order $\leq k$ and their closeness have been defined then the closeness $cl_p(G, G')$ (at least in degree p) between granules G, G' of order $k+1$ can be defined by applying an appropriate operator F to closeness values of components of G, G', respectively.

A general scheme of defining more complex granule from simpler ones can be explored using rough mereological approach [2].

Inclusion and closeness of elementary granules. We have introduced the simplest case of granules in information system $IS = (U, A)$. They are defined by $EF_B(x)$, where EF_B is a conjunction of selectors of the form $a = a(x)$, $B \subseteq A$ and $x \in U$. Let $G_{IS} = \{EF_B(x) : B \subseteq A \ \& \ x \in U\}$. In the standard rough set model [1] elementary granules describe indiscernibility classes with respect to some subsets of attributes. In a more general setting see e.g. [3], [5] tolerance (similarity) classes are described.

The crisp inclusion of α in β, where $\alpha, \beta \in \{EF_B(x) : B \subseteq A \ \& \ x \in U\}$ is defined by $\|\alpha\|_{IS} \subseteq \|\beta\|_{IS}$, where $\|\alpha\|_{IS}$ and $\|\beta\|_{IS}$ are sets of objects from IS satisfying α and β, respectively. The non-crisp inclusion, known in KDD, for the case of association rules is defined by means of two thresholds t and t' :

$support_{IS}(\alpha, \beta) = card(\|\alpha \wedge \beta\|_{IS}) \geq t$, and
$accuracy_{IS}(\alpha, \beta) = \frac{support_{IS}(\alpha,\beta)}{card(\|\alpha\|_{IS})} \geq t'$.

Elementary granule inclusion in a given information system IS can be defined using different schemes, e.g., by

$\nu^{IS}_{t,t'}(\alpha, \beta)$ if and only if $support_{IS}(\alpha, \beta) \geq t$ & $accuracy_{IS}(\alpha, \beta) \geq t'$.

The closeness of granules can be defined by

$cl^{IS}_{t,t'}(\alpha, \beta)$ if and only if $\nu^{IS}_{t,t'}(\alpha, \beta)$ and $\nu^{IS}_{t,t'}(\beta, \alpha)$ hold.

Decision rules as granules. One can define inclusion and closeness of granules corresponding to rules of the form **if** α **then** β using accuracy coefficients.

Having such granules $g = (\alpha, \beta)$, $g' = (\alpha', \beta')$ one can define inclusion and closeness of g and g' by $\nu_{t,t'}(g, g')$ if and only if $\nu_{t,t'}(\alpha, \alpha')$ and $\nu_{t,t'}(\beta, \beta')$.

The closeness can be defined by

$cl_{t,t'}(g, g')$ if and only if $\nu_{t,t'}(g, g')$ and $\nu_{t,t'}(g', g)$.

Extensions of elementary granules by tolerance relation. For extensions of elementary granules defined by similarity (tolerance) relation, i.e., granules of the form (α, τ), (β, τ) one can consider the following inclusion measure:

$\nu^{IS}_{t,t'}((\alpha, \tau)(\beta, \tau))$ if and only if

$\nu^{IS}_{t,t'}(\alpha', \beta')$ for any α', β' such that $(\alpha, \alpha') \in \tau$ and $(\beta, \beta') \in \tau$

and the following closeness measure:

$cl^{IS}_{t,t'}((\alpha, \tau)(\beta, \tau))$ if and only if $\nu^{IS}_{t,t'}((\alpha, \tau)(\beta, \tau))$ and $\nu^{IS}_{t,t'}((\beta, \tau)(\alpha, \tau))$.

Sets of rules. It can be important for some applications to define closeness of an elementary granule α and the granule (α, τ). The definition reflecting an intuition that α should be a representation of (α, τ) sufficiently close to this granule is the following one:

$cl^{IS}_{t,t'}(\alpha, (\alpha, \tau))$ if and only if $cl_{t,t'}(\alpha, \beta)$ for any $(\alpha, \beta) \in \tau$.

An important problem related to association rules is that the number of such rules generated even from simple data table can be large. Hence, one should search for methods of aggregating close association rules. We suggest that this can be defined as searching for some close information granules.

Let us consider two finite sets $Rule\_Set$ and $Rule\_Set'$ of association rules defined by $Rule\_Set = \{(\alpha_i, \beta_i) : i = 1, \ldots, k\}$, and
$Rule\_Set' = \{(\alpha'_i, \beta'_i) : i = 1, \ldots, k'\}$. One can treat them as higher order information granules. These new granules $Rule\_Set$, $Rule\_Set'$ can be treated as close in a degree at least t (in IS) if and only if there exists a relation rel between sets of rules $Rule\_Set$ and $Rule\_Set'$ such that:

1. For any $Rule$ from the set $Rule\_Set$ there is $Rule'$ from $Rule\_Set'$ such that $(Rule, Rule') \in rel$ and $Rule$ is close to $Rule'$ (in IS) in degree at least t.
2. For any $Rule'$ from the set $Rule\_Set'$ there is $Rule$ from $Rule\_Set$ such that $(Rule, Rule') \in rel$ and $Rule$ is close to $Rule'$ (in IS) in degree at least t.

Another way of defining closeness of two granules G_1, G_2 represented by sets of rules can be described as follows.

Let us consider again two granules $Rule\_Set$ and $Rule\_Set'$ corresponding to two decision algorithms. By $I(\beta'_i)$ we denote the set $\{j : cl_p(\beta'_j, \beta'_i)\}$ for any $i = 1, \ldots, k'$.

Now, we assume $\nu_p\left(Rule\_Set, Rule\_Set'\right)$ if and only if for any $i \in \{1, \ldots, k'\}$ there exists a set $J \subseteq \{1, \ldots, k\}$ such that

$$cl_p\left(\bigvee_{j \in I(\beta_i')} \beta_j', \bigvee_{j \in J} \beta_j\right) \text{ and } cl_p\left(\bigvee_{j \in I(\beta_i')} \alpha_j', \bigvee_{j \in J} \alpha_j\right)$$

and for closeness we assume

$cl_p\left(Rule\_Set, Rule\_Set'\right)$ if and only if

$\nu_p\left(Rule\_Set, Rule\_Set'\right)$ and $\nu_p\left(Rule\_Set', Rule\_Set\right)$.

One can consider a searching problem for a granule $Rule\_Set'$ of minimal size such that $Rule\_Set$ and $Rule\_Set'$ are close.

Granules defined by sets of granules. The previously discussed methods of inclusion and closeness definition can be easily adopted for the case of granules defined by sets of already defined granules. Let G, H be sets of granules.

The inclusion of G in H can be defined by

$\nu_{t,t'}^{IS}(G, H)$ if and only if for any $g \in G$ there is $h \in H$ for which $\nu_{t,t'}^{IS}(g, h)$ and the closeness by $cl_{t,t'}^{IS}(G, H)$ if and only if $\nu_{t,t'}^{IS}(G, H)$ and $\nu_{t,t'}^{IS}(H, G)$.

We have the following examples of inclusion and closeness propagation rules:

$$\frac{\text{for any } \alpha \in G \text{ there is } \alpha' \in H \text{ such that } \nu_p(\alpha, \alpha')}{\nu_p(G, H)}$$

$$\frac{cl_p(\alpha, \alpha'), cl_p(\beta, \beta')}{cl_p((\alpha, \beta), (\alpha', \beta'))}$$

$$\frac{\text{for any } \alpha' \in \tau(\alpha) \text{ there is } \beta' \in \tau(\beta) \text{ such that } \nu_p(\alpha', \beta')}{\nu_p((\tau : \alpha), (\tau : \beta))}$$

$$\frac{cl_p(G, G') \text{ and } cl_p(E, E')}{cl_p((G, E), cl_p(G', E'))}$$

where $\alpha, \alpha', \beta, \beta'$ are elementary granules and G, G' are finite sets of elementary granules.

One can also present other discussed cases for measuring the inclusion and closeness of granules in the form of inference rules. The exemplary rules have a general form, i.e., they are true in any information system (under the chosen definition of inclusion and closeness).

4 Approximations of information granule sets

We introduce now the approximation operations for granule sets assuming a given granule system $\mathcal{G}$ specified by syntx, semantics of information granules from the universe U as well as by the relations of inclusion ν_p and closeness cl_q in degrees at least p, q, respectively.

For a given granule g we define its neighborhood $I_p(g)$ to be the set of all information granules from U close to g in degree at least p.

For any subset $X \subseteq U$ we define its lower and upper approximation by

$LOW(\mathcal{G}, p, q, X) = \{g \in U : \nu_q(I_p(g), X)\}$,

$UPP(\mathcal{G}, p, t, X) = \{g \in U : \nu_t(I_p(g), X)\}$, respectively.

where $\nu_q(I_p(g), X)$ iff for any granule $r \in I_p(g)$ the condition $\nu_q(r, x)$ holds for some $x \in X$ and $0.5 < t < q$.

Hence it follows that the approximations of sets can be defined analogously to the classical rough set approach. In our next paper we will discuss how to define approximation of complex information granules taking into account their structure (e.g., defined by the relation *to be a part in a degree* [2]).

Conclusions

We have presented the concept of approximation of complex information granule sets. This notion seems to be crucial for further investigations on approximate reasoning based on information granules.

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References

1. Pawlak Z.: Rough Sets. Theoretical Aspects of Reasoning about Data, Kluwer Academic Publishers, Dordrecht, 1991.
2. Polkowski L., Skowron A.: Towards Adaptive Calculus of Granules, In: [8], vol.1, pp. 201–227.
3. Skowron A., Stepaniuk J.: Tolerance Approximation Spaces, Fundamenta Informaticae, Vol. 27, 1996, pp. 245–253.
4. Skowron A., Stepaniuk J.: Information Granules in Distributed Environment, Lecture Notes in Artificial Intelligence 1711, Springer-Verlag, 1999, pp. 357–365.
5. Stepaniuk J.: Knowledge Discovery by Application of Rough Set Models, ICS PAS Report 887, Institute of Computer Science, Polish Academy of Sciences, Warsaw 1999, and also: L. Polkowski, T.Y. Lin, S. Tsumoto (Eds.), Rough Sets: New Developments, Physica–Verlag, Heidelberg, 2000.
6. WWW SPACENET page: http://agora.scs.leeds.ac.uk/spacenet/.
7. Zadeh L.A.: Fuzzy Logic = Computing with Words, IEEE Trans. on Fuzzy Systems Vol. 4, 1996, pp. 103–111.
8. Zadeh L.A., Kacprzyk J. (Eds.): Computing with Words in Information/Intelligent Systems vol.1–2, Physica–Verlag, Heidelberg, 1999.