Rough Sets: Trends and Challenges
Extended Abstract

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Abstract. We discuss how approximation spaces considered in the context of rough sets and information granule theory have evolved over the last 20 years from simple approximation spaces to more complex spaces. Some research trends and challenges for the rough set approach are outlined in this paper. The study of the evolution of approximation space theory and applications is considered in the context of rough sets introduced by Zdzisław Pawlak and the notions of information granulation and computing with words formulated by Lotfi Zadeh. The deepening of our understanding of information granulation and the introduction to new approaches to concept approximation, pattern identification, pattern recognition, pattern languages, clustering, information granule systems, and inductive reasoning have been aided by the introduction of a calculus of information granules based on rough mereology. Central to rough mereology is the inclusion relation to be a part to a degree. This calculus has grown out of an extension of what S. Leśniewski called mereology (the study of what it means to be a part of).

1 Introduction

One of the basic concepts of rough set theory [18] is the indiscernibility relation defined by means of information about objects of interest. The indiscernibility relation is used to define set approximations [17,18].

Several generalizations of the rough set approach based on approximation spaces defined by tolerance and similarity relation or a family of indiscernibility relations, have been reported (for references see the papers and bibliography in [16,23]). Rough set approximations have been also generalized for preference relations and rough-fuzzy hybridizations (see, e.g., [39]). Generalized approximation spaces are discussed in [35] where uncertainty and inclusion functions are introduced. The approach based on inclusion functions has been generalized to the *rough mereological approach* (see, e.g., [22,24,25]). The inclusion relation $x\mu_r y$ with the intended meaning *x is a part of y to a degree r* has been taken as the basic notion of the rough mereology which is a generalization of the Leśniewski mereology [8].

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In the following sections we will discuss the impact of information granulation and inductive reasoning on the concept approximation process.

As a result of inductive reasoning one cannot define inclusion degrees of object neighborhoods directly into the target concepts but only into some patterns relevant to such concepts (e.g., left hand sides of decision rules) (see, e.g., [30], [44], [38], [2], [33]). Such degrees together with degrees of inclusion of patterns in target concepts make it possible to define outputs of classifiers for new classified objects. Using the constructed classifiers one can define new patterns relevant to concept approximation [2,33]. The research in this direction has been recently initiated. It can bring new interesting results related to the rough set approach in inductive reasoning, in particular for adaptive learning of concepts.

Let us note that the rough set approach for more complex data such as decision tables with transformations (describing deformations of objects) preserving classification of objects, complex decisions or attribute values (e.g., being plans or models of processes) should be developed. Some issues of the rough set approach concerned with data and domain knowledge represented in distributed environments are outlined in the following sections.

2 Rough-Mereological Approach to Approximation of Information Granules

Rough mereology offers a methodology for synthesis and analysis of objects in the distributed environments of intelligent agents, in particular, for synthesis of objects satisfying a given specification to a satisfactory degree or for control in such complex environments. Moreover, rough mereology has been recently used for developing foundations of the *information granule calculus*, an attempt towards formalization of the paradigm of computing with words based on perception recently formulated by Lotfi Zadeh [45,46,47].

The rough mereological approach built on the basis of the inclusion relation *to be a part to a degree* generalizes the rough set and fuzzy set approaches (see. e.g., [22], [24], [25], [26]). Such inclusion relations, called *rough inclusions*, can be used to define other basic concepts like closeness of information granules, their semantics, indiscernibility and discernibility of objects, information granule approximation and approximation spaces, perception structure of information granules as well as the notion of ontology approximation. For details the reader is referred to [15]. The rough inclusion relations together with operations for construction new information granules from already existing ones create a core of a calculus of information granules.[1] Distributive multi-agent framework makes it possible to create a relevant computational model for calculus of information granules. Agents (information sources) provide us with information granules that must be transformed, analyzed and built into structures that support problem solving. In such computational model approximation spaces play an important

[1] Note, the rough inclusion relations should be extended on newly constructed information granules.

role because information granules received by agents must be approximated (to be understandable by them) before they can be transformed (see, e.g., [24], [36], [15], [21]). [2]

Developing calculi of information granules for approximate reasoning is a challenge important for many applications including control of autonomous vehicles [43] and line-crawling robots [19], web mining and spatio-temporal data mining [30], design automation, sensor fusion [20], approximation neuron design [21,15], creating approximate views of relational databases and, in general, for embedding in intelligent systems ability for reasoning with words and reasoning based on perception [45,46,47]. Some steps towards this direction have been taken. Methods for construction of approximate reasoning schemes (*AR*-schemes) have been developed. Such *AR*-schemes are information granules being clusters of exact constructions (derivations). Reasoning with *AR*-schemes makes it possible to obtain results satisfying a given specification up to a satisfactory degree (not necessarily exactly) (see e.g., [24,15,30,31]). Methods based on hybridization of rough sets with fuzzy sets, neural networks, evolutionary approach or case based reasoning are especially valid in inducing *AR*-schemes.

Let us finally note that inducing relevant calculi of information granules includes also such complex tasks like discovery of relevant operations on information granules or rough inclusion measures. This is closely related to problems of perception and reasoning based on perception [47].

Using rough inclusions, one can generalize the approximation operations for sets of objects, known in rough set theory, to arbitrary information granules. The approach is based on the following reasoning:

Assume p is an inclusion degree, $G = \{g_t\}_t$ is a given family of information granules and g is a granule from a given information granules system S. Let us recall that inclusion degrees are partially ordered by a relation $\leq$. Now, assuming $p < q$, one can consider two approximations for a given information granule g by G. The (G, q)-*lower approximation* of g is defined by [3]

$$LOW_{G,p,q}(g) = Make\_granule(\{g_t : \nu_{q'}(g_t, g) \text{ and } q' \geq q\}). \quad (1)$$

The (G, q)-*upper approximation* of g is defined by

$$UPP_{G,p,q}(g) = Make\_granule(\{g_t : \nu_{p'}(g_t, g) \text{ and } p' > p\}). \quad (2)$$

The definition of a parameterized approximation space given in [35] is an example of the introduced notion of information granule approximation. The presented approach can be generalized to approximation spaces in inductive reasoning.

[2] Recently, relationships between the rough set approach [37] and the information flow approach to logic of distributed systems [1] have been reported.

[3] $Make\_granule$ operation is a fusion operation of collections of information granules. A typical example of $Make\_granule$ is set theoretical union used in rough set theory. Another example of $Make\_granule$ operation is realized by classifiers.

3 Rough Sets and Inductive Reasoning

In inductive reasoning we would like to approximate concepts over a universe of objects, say U^∞, wider than the universe U of objects in a given decision system. In other words, assuming $U \subset U^\infty$, we would like to approximate concepts over U^∞ which are extensions of decision classes in a given decision system. In this section, we present the relevant approximation spaces for such concepts, and show how to induce *classifiers* approximating those concepts. We also discuss the relationships between the whole process and different approaches pursued in the fields like machine learning, pattern recognition, data mining and knowledge discovery [10,6,29].

The main observation is that, in the considered case, it is necessary to induce also a relevant approximation space. Such a space is usually different from the partition defined by the conditional attributes of a given decision system. It consists of some subsets of U^∞, called neighborhoods of objects. It should be emphasized that neighborhoods usually create a covering of U^∞, not necessarily a partition. They are defined by *patterns* chosen from some relevant *pattern languages*. In practical applications it is often necessary to specify a data model using a particular description in a pattern language. Moreover, the description usually is consistent only on a given part of the model, since the whole original model is often only partially specified. [4] In order to indicate that a given model is specified by a particular description, we use the term *description model*.

The structure of the pattern languages and the patterns themselves should be discovered. The whole process is quite complex and is illustrated in Fig. 1, where: $\mathcal{A} = (U, A, d)$ denotes a decision system; $\mathcal{A}_{train}$ and $\mathcal{A}_{test}$ are training and testing subsystems of $\mathcal{A}$, respectively; $\mathcal{L} = \{L_i\}_{i \in I}$ is a family of pattern languages; $\mathcal{Q} = \{Q_j\}_{j \in J}$ is a family of quality measures for description models; M is a description model covering objects in U; C is a classifier obtained from M and covering the (almost) whole universe U^∞.

Elements of L_i are formulas called *patterns*. Patterns define, in a given decision system, sets of objects in which they are satisfied. Description models describe decision classes of $\mathcal{A}$, by using patterns from L_i and some inclusion measures of those patterns in decision classes. The description models can be built by means of, e.g., decision rules over descriptors from L_i.

Quality measures can be used as criteria for tuning the model. For a given L_i and Q_j, one can search for a description model using patterns from L_i which is (sub-) optimal with respect to the measure Q_j. However, the goal is to induce the relevant description model for the induced classifier, covering the whole universe of objects.

This, in particular, makes it necessary to tune parameters of the description quality measure. There are many ways for specifying quality measures. For example, a measure Q_j, can be specified using the *minimum description length principle* [27,40], where one estimates the quality of approximation, as well as the size of the description model defined. The minimum description length principle

[4] We will discuss this issue in more detail later in this section.

requires to choose a description of the smallest size from among those descriptions with the same approximation quality. In this case, the quality measure depends on two arguments. The first argument represents the quality of approximation (e.g., using the positive region of decision classes or entropy measure). The second argument represents the measures based on the model size. A proper balance between these two arguments is generally obtained using training data. The tuning may involve thresholds for degrees of inclusion of patterns from L_i in decision classes or for the positive region size.

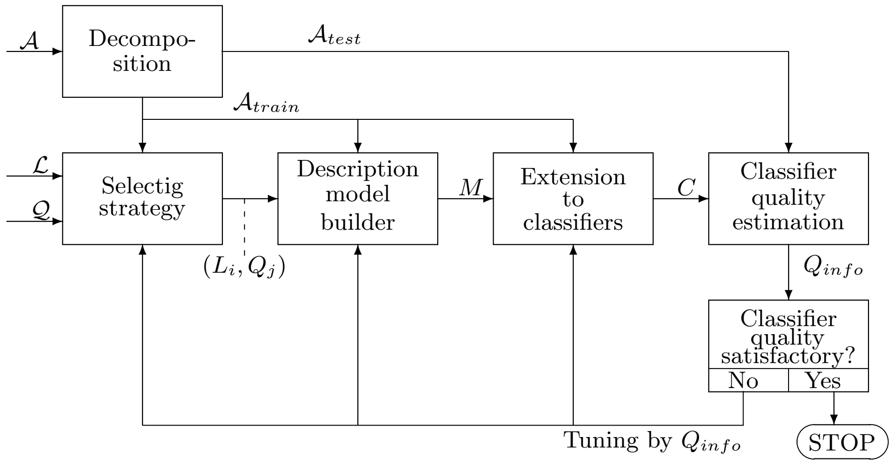

Fig. 1. Approximation space and classifier construction using rough sets.

The whole process, presented in Fig. 1, can be viewed as a process searching for a relevant approximation space. As we have mentioned before, such an approximation space consists of neighborhoods of objects from U. Certainly, such an approximation space is more general than what is discussed in [18].

The induced description model should be extended to a classifier of all objects from the whole universe of objects U^∞, not only from U (the reader is referred, e.g., to [10], [15] for the definition of classifiers). Recall that for any object to be classified, it is necessary to compute its degree of inclusion in any pattern from the description model. In the case of new objects (outside of U), these degrees can suggest conflicting decisions and, together with the degrees of pattern inclusion in decision classes, create input for conflict resolution strategy necessary to compute the classifier output.

Next, the induced classifier is tested on objects from $\mathcal{A}_{test}$. Information Q_{info} about the classifier behavior is returned from the classifier quality estimation module. If Q_{info} shows that the classifier quality is unsatisfactory, it is used to tune parameters in different modules presented in Fig. 1 and to reconstruct the classifier to obtain a new one with a better quality. In addition, matching strategies for objects and patterns as well as parameters for conflict resolution strategy can also be tuned. The parameters involved in the tuning process can, for instance, be inclusion degree thresholds, parameters characterizing approximation quality or parameters measuring the description model size.

As a typical example, one can consider the language of patterns consisting of conjunctions of descriptors over a selected set of attributes. More complex pattern language can include conjunctions of formulas that are disjunctions of descriptor conjunctions.

The induced approximation spaces can be treated as an example of complex information granules.

3.1 Classifiers

An important class of information granules create classifiers, i.e., algorithms classifying objects into decision classes. The classifier construction based on approximation space induced from a given decision table $DT = (U, A, d)$ can be described as follows [30]:

1. First, one can construct granules G_j corresponding to each particular decision $j = 1, \ldots, r$ by taking a collection $\{g_{ij} : i = 1, \ldots, k_j\}$ of left hand sides of decision rules for a given decision.
2. Let E be a set of elementary granules (e.g., defined by conjunction of descriptors) over $\mathcal{A} = (U, A)$. We can now consider a granule denoted by $Match(e, G_1, \ldots, G_r)$ for any $e \in E$ being a collection of coefficients ε_{ij} where $\varepsilon_{ij} = 1$ if the set of objects defined by e in $\mathcal{A}$ is included in the meaning of g_{ij} in $\mathcal{A}$, i.e., $Sem_{\mathcal{A}}(e) \subseteq Sem_{\mathcal{A}}(g_{ij})$; and 0, otherwise. Hence, the coefficient ε_{ij} is equal to 1 if and only if the granule e matches in $\mathcal{A}$ the granule g_{ij}.
3. Let us now denote by $Conflict\_res$ an operation (resolving conflict between decision rules recognizing elementary granules) defined on granules of the form $Match(e, G_1, \ldots, G_r)$ with values in the set of possible decisions $1, \ldots, r$. Hence, $Conflict\_res(Match(e, G_1, \ldots, G_r))$ is equal to the decision predicted by the classifier $Conflict\_res(Match(\bullet, G_1, \ldots, G_r))$ on the input granule e.

Parameters to be tuned in classifiers are voting strategies, matching strategies of objects against rules as well as other parameters like closeness of granules in the target granule.

The reader can easily describe more complex classifiers by means of information granules. For example, one can consider soft instead of crisp inclusion between elementary information granules representing classified objects and the left hand sides of decision rules or soft matching between recognized objects and left hand sides of decision rules.

One can use the constructed classifiers in searching for a new approximation space relevant for the target concept approximation on the universe of objects extended by the testing objects. New neighborhoods in such approximation space are defined by parameterized patterns expresses in a language used for classifier construction (e.g., one can consider parameterized patterns defined by formulas used for conflict resolution between voting decision rules [2]). The relevant patterns and hence the neighborhoods for concept approximation can be obtained by tuning parameters of such parameterized patterns. The new patterns can also be used in adaptive reconstruction of classifiers. Developing rough set based strategies for adaptive concept approximation is a challenge. Some results on rough set approach to adaptive classifier construction are reported [44].

4 Rough Sets, Boolean Reasoning, and Approximate Boolean Reasoning

In this section we discuss a methodology that makes it possible to search for patterns defining neighborhoods of approximation spaces relevant for concept approximation. The ability to discern between perceived objects is important for constructing many different kinds of reducts making possible to define relevant patterns for concepts approximations.

The idea of Boolean reasoning is based on construction for a given problem P a corresponding Boolean function f_P with the following property: the solutions for the problem P can be decoded from prime implicants of the Boolean function f_P [4]. Let us mention that to solve real-life problems it is necessary to deal with Boolean functions having a large number of variables.

A successful methodology based on the discernibility of objects and Boolean reasoning has been developed for computing of many different kinds of reducts and their approximations for inducing decision rules, association rules, discretization of real value attributes, symbolic value grouping, searching for new features defined by oblique hyperplanes or higher order surfaces, pattern extraction from data as well as conflict resolution or negotiation (for references see, e.g., [7], [29], [11], [12], [32], [40] and bibliography in [16], [23], [7], [41]). Most of the problems related to generation of the above mentioned entities are (at least) NP-complete or NP-hard. However, it was possible to develop efficient heuristics returning suboptimal solutions of the problems. The results of experiments on many data sets are very promising. They show very good quality of solutions generated by the heuristics in comparison with other methods reported in literature (e.g. with respect to the classification quality of unseen objects). Moreover, they are very efficient from the point of view of time necessary for computing of the solution. It is important to note that the methodology allows for some problems to construct heuristics having a very important *approximation property* which can be formulated as follows: expressions generated by heuristics (i.e., implicants) *close* to prime implicants define approximate solutions for the problem.

A challenging issue is to develop a methodology called *approximate Boolean reasoning* for deriving such heuristics feasible for a wide class of problems related to rough set applications, e.g., in data mining. Such an approach is also suggested in [28]. However, the problems we are dealing with require analysis of very large formulas for which general purpose heuristics will not be feasible. One possibility is to develop heuristics feasible for such problems is to use domain knowledge about them. Some promising results in this direction have been obtained [11,12, 7,29].

5 Learning from Sparse Data

One of the main issue in learning theory is to develop methodology for reasoning from *sparse data* [42,3]. This research direction been recently suggested by statisticians as a new direction not based on searching for stochastic data models generating them. Developing methods based on rough sets for reasoning from sparse data is a challenge for the rough set approach. Some results in this direction are reported [5], [7], [23]. However, much more work should be done in this direction. For example, evolutionary strategies discovering subspaces of features (from which relevant features can be selected) should be developed. Such strategies can be gained from experience with learning systems that search for such subspaces. We have suggested that in searching for such strategies domain knowledge can also be used. Then inducing productions from data becomes feasible because, in a sense, there is a *sufficiently small distance* between feature spaces of premisses and conclusions of such rules. Next AR-schemes can be derived from productions that have been discovered [24,30,36,33].

6 Conclusions

We have discussed different aspects of approximation spaces and some current research trends and challenges related to concept approximation issues. Among these trends and challenges are those related to (i) approximate Boolean reasoning, (ii) inducing operations on information granules, inclusion and closeness measures, productions and AR-schemes, (iii) domain knowledge approximation, (iv) reasoning from sparse data, (v) adaptive learning of concepts, (vi) computational models for calculi on information granules, (vii) rough-neuro computing. Many of these trends and challenges are closely related to computing with words and computational theory of perceptions [47]. All of them are concentrated around searching for calculi of information granules for approximate reasoning. We have shown that approximation spaces developed and investigated in rough set theory play important role in all discussed research directions.

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