Rough Sets and Systems of Granules

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Abstract

The aim of the paper is to present an outline of granular computing framework for spatial reasoning. In our previous papers we have discussed basic notions related to granular computing, namely the information granule syntax and semantics as well as the inclusion and closeness (similarity) relations of granules. Different information sources (units, agents) are equipped with two kinds of operations on information granules: operations possessed by agents transforming tuples of information granules into new granules and approximation operations for computing by agents information granule approximations delivered by other agents. More complex granules are constructed by means of these operations from some input information granules.

1 Motivation

We would like to discuss briefly an example showing a motivation for our work [14]. Let us consider a team of agents recognizing the situation on the road. The aim is to classify a given situation as, e.g., *dangerous or not*. This *soft specification granule* is represented by a family of information granules called *case soft patterns* representing cases, like *cars are too close*. The whole scene (actual situation on the road) is decomposed into regions perceived by local agents. Higher level agents can reason about regions observed by team of their children agents. They can express in their own languages features used by their children. Moreover, they can use new features like attributes describing relations between regions perceived by children agents. The problem is how to organize agents into a team (having, e.g., tree structure) with the property that the information granules synthesized by the team from input granules (being perceptions of local agents from sensor measurements) will identify the situation on the road in the following sense: the granule constructed by the team from input granules representing the situation on the road is sufficiently close to the soft specification granule named *dangerous* if and only if the situation on the road is really dangerous. We expect that if the team is returning a granule sufficiently close to the soft specification granule *dangerous* then also a special case of the soft pattern *dangerous* is identified helping to explain the situation.

The aim of our project is to develop foundations for this kind of reasoning. In particular it is necessary to give precise meaning to the notions like: information granules, soft information granules, closeness of information granules in satisfactory degree, information granules synthesized by team of agents etc. The presented paper realizes the first step toward this goal.

The general scheme is depicted in Figure 1.

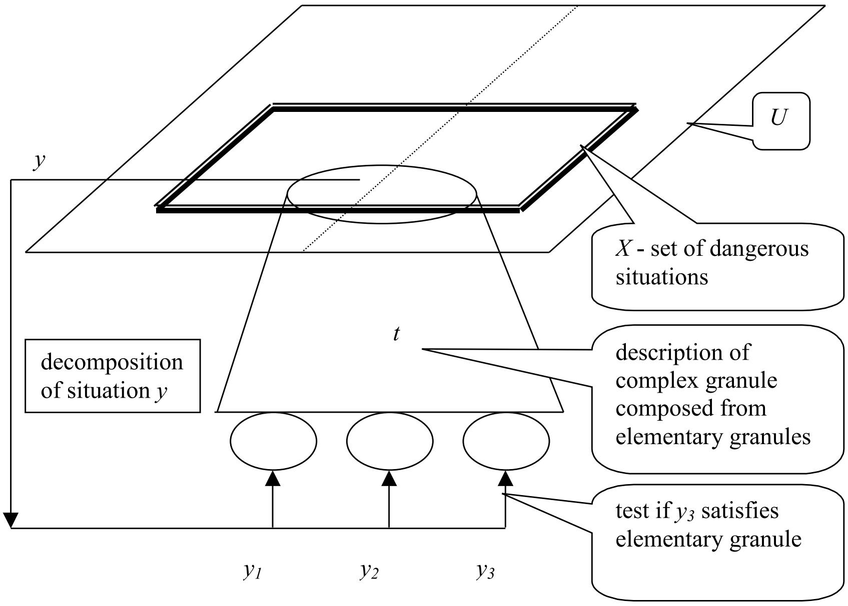

Figure 1: Illustrative Example

To sum up, we consider a set of agents Ag. Each agent is equipped with some approximation spaces (defined using rough set approach [3]). Agents are cooperating to solve a problem specified by a special agent called *customer-agent*. The result of cooperation is a scheme of agents. In the simplest case the scheme can be represented by a tree labeled by agents. In this tree leaves are delivering some information granules (representing of perception in a given situation by leaf agents) and any non-leaf agent $ag \in Ag$ is performing an operation $o(ag)$ on approximations of granules delivered by its children. The root agent returns an information granule being the result of computation by the scheme on granules delivered by leaf agents. It is important to note that different agents use different languages. Thus granules delivered by children agents to their father can be usually perceived by him in an approximate sense before he can perform any operation on delivered granules.

2 Inroduction to Information Granules

Methods for qualitative spatial reasoning [5], [2], [13], [1], [14] are closely related to a paradigm Computing with Words recently formulated by Lotfi Zadeh [15], [16]. Several attempts have been made to develop foundations for computing with words [16]. Among them there is a rapidly growing area of granular computing aiming to develop models for computing with information granules (see e.g. [4]).

They are two basic notions for granular computing: information granule and calculus on information granules [4].

Notions of information granule [15], [4] and information granule similarity (inclusion or closeness) are very useful for knowledge discovery. Informally s-peaking, information granules can be treated as linked collections of objects drawn together by the criteria of indiscernibility, similarity or functionality [15].

In [7], [8] several examples of complex information granules have been discussed. We have presented syntax, semantics, relations of inclusion ν_p and closeness cl_p for information granules and a general recursive scheme for construction of more complex granules from simpler ones. In particular, the inclusion and closeness relations for more complex granules are defined by extension of these relations for the granules being parts of those complex granules.

In this paper we elaborate a general scheme for information granule construction in distributed systems introduced in [8].

Teams of agents are organized, e.g., along the schemes of decomposition of complex objects (representing situations on the road) into trees. The trees are represented by expressions called terms. Two granules are defined being values of t under the valuation val for any valuation val of leaf agents of a given term t in the set of input granules. They are called the lower and upper approximations of t under val. The necessity to consider rather approximation of granule returned by a given term t under a given valuation val than the exact value of t under val is a consequence of the mentioned above ability of agents to perceive in approximate sense only of information granules received from other agents. Similarity relations extracted from data allow to measure the closeness of these granules, in particular to the soft specification granule.

We consider problems of agent team (terms) synthesis for different tasks. For example, we are looking for a strategy returning for any valuation val (representing global situation) a term (agent team) t with the following property: the lower and upper values of t under val are sufficiently close to a given soft specification granule if and only if the global situation represented by val really matches this specification.

We also emphasize [9] the problem of the robust granule construction. We use some ideas from rough mereology [4] to specify the rules describing the ranges in which parameters of granules being arguments of operations on granules can be changed to assure that the results of the operations on these granules are sufficiently close. We suggest that such rules should be extracted from data. The construction of such robust granules seems to be important for spatial reasoning.

Progress in solving the above discussed problems is strongly dependent on further development of foundations of granular computing including soft information granule understanding, methods for measuring of different kinds of information granule closeness or methods for information granule transformation.

In the sequel we discuss one of the basic approximation problem. For given complex granule and a set of information granules we would like to define up to what degree this complex granule can be approximated by the given set of granules.

First we present approximation spaces and rough sets.

Next we present some examples of information granules and we extend the classical rough set approach for object set approximation to complex granules.

3 Rough Sets and Approximation Spaces

We recall general definition of approximation space [6], [11].

Definition 1 *A parameterized approximation space is a system* $AS_{\#,\$} = (U, I_{\#}, \nu_{\$})$*, where*

- U *is a non-empty set of objects,*
- $I_{\#} : U \rightarrow P(U)$*, where* $P(U)$ *denotes the powerset of* U*, is an uncertainty function,*
- $\nu_{\$} : P(U) \times P(U) \rightarrow [0,1]$ *is a rough inclusion function.*

The uncertainty function defines for every object x a set of similarly described objects. A constructive definition of uncertainty function can be based on the assumption that some metrics (distances) are given on attribute values. For example, if for some attribute $a \in A$ a metric $\delta_a : V_a \times V_a \longrightarrow [0,\infty)$ is given, where V_a is the set of all values of attribute a, then one can define the following uncertainty function:

$$y \in I_a^{f_a}(x) \text{ if and only if } \delta_a(a(x), a(y)) \leq f_a(a(x), a(y)),$$

where $f_a : V_a \times V_a \rightarrow [0,\infty)$ is a given threshold function.

A set $X \subseteq U$ is *definable in* $AS_{\#,\$}$, if it is a union of some values of the uncertainty function.

The rough inclusion function defines the degree of inclusion between two subsets of U [6], [4].

Thee lower and the upper approximations of subsets of U are defined as follows.

Definition 2 *For a parameterized approximation space* $AS_{\#,\$} = (U, I_{\#}, \nu_{\$})$ *and any subset* $X \subseteq U$ *the lower and the upper approximations are defined by*

$LOW\left(AS_{\#,\$}, X\right) = \{x \in U : \nu_{\$}\left(I_{\#}\left(x\right), X\right) = 1\}$,

$UPP\left(AS_{\#,\$}, X\right) = \{x \in U : \nu_{\$}\left(I_{\#}\left(x\right), X\right) > 0\}$, *respectively.*

Approximations of concepts (sets) are constructed on the basis of background knowledge. Obviously, concepts are also related to new (unseen) objects. Hence it is very useful to define parameterized approximations with parameters tuned in the searching process for approximations of concepts. This idea is crucial for construction of concept approximations using rough set methods. In our notation $\#, \$$ denote vectors of parameters which can be tuned in the process of concept approximation.

4 Examples of information granules

Usually, together with approximation space, there is also specified a set of formulas Φ expressing properties of objects. Hence, we assume that together with the approximation space $AS_{\#,\$}$ there are given

- a set of formulas Φ over some language,
- semantics $\|\cdot\|$ of formulas from Φ, i.e., a function from Φ into the power set $P(U)$.

Let us consider an example [3]. We define a language L_{IS} used for elementary granule description, where $IS = (U, A)$ is an information system. The syntax of L_{IS} is defined recursively by

1. $(a \in V) \in L_{IS}$, for any $a \in A$ and $V \subseteq V_a$.
2. If $\alpha \in L_{IS}$ then $\neg\alpha \in L_{IS}$.
3. If $\alpha, \beta \in L_{IS}$ then $\alpha \wedge \beta \in L_{IS}$.
4. If $\alpha, \beta \in L_{IS}$ then $\alpha \vee \beta \in L_{IS}$.

The semantics of formulas from L_{IS} with respect to an information system IS is defined recursively by

1. $\|a \in V\|_{IS} = \{x \in U : a(x) \in V\}$.
2. $\|\neg\alpha\|_{IS} = U - \|\alpha\|_{IS}$.
3. $\|\alpha \wedge \beta\|_{IS} = \|\alpha\|_{IS} \cap \|\beta\|_{IS}$.
4. $\|\alpha \vee \beta\|_{IS} = \|\alpha\|_{IS} \cup \|\beta\|_{IS}$.

A typical method used by the classical rough set approach [3] for constructive definition of the uncertainty function is the following: for any object $x \in U$, there is given information $Inf_A(x)$ (information signature of x relatively to A) which can be interpreted as a conjunction $EF_B(x)$ of selectors $a = a(x)$ for $a \in A$ and the set $I_{\#}(x)$ is equal to $\|EF_B(x)\|_{IS} = \left\|\bigwedge_{a \in A} a = a(x)\right\|_{IS}$. One can consider a more general case taking as possible values of $I_{\#}(x)$ any set $\|\alpha\|_{IS}$ containing x. Next from the family of such sets the resulting neighborhood $I_{\#}(x)$ can be selected or constructed. One can also use another approach by considering more general approximation spaces in which $I_{\#}(x)$ is a family of subsets of U.

In the sequel we will consider several examples of information granule constructions. We present now the syntax and semantics of information granules. In the following section we discuss the inclusion and closeness relations for granules.

Elementary granules. In an information system $IS = (U, A)$, elementary granules are defined by $EF_B(x)$, where EF_B is a conjunction of selectors of the form $a = a(x)$, $B \subseteq A$ and $x \in U$. For example, the meaning of an elementary granule $a = 1 \wedge b = 1$ is defined by

$$\|a = 1 \wedge b = 1\|_{IS} = \{x \in U : a(x) = 1 \ \& \ b(x) = 1\}.$$

Sequences of granules. Let us assume that S is a sequence of granules and the semantics $\|\bullet\|_{IS}$ in IS of its elements have been defined. We extend $\|\bullet\|_{IS}$ on S by $\|S\|_{IS} = \{\|g\|_{IS}\}_{g \in S}$.

Example 3 *Granules defined by rules in information systems are examples of sequences of granules. Let IS be an information system and let (α, β) be a new information granule received from the rule* ***if*** α ***then*** β *where α, β are elementary granules of IS. The semantics $\|(\alpha, \beta)\|_{IS}$ of (α, β) is the pair of sets $(\|\alpha\|_{IS}, \|\beta\|_{IS})$.*

Sets of granules. Let us assume that a set G of granules and the semantics $\|\bullet\|_{IS}$ in IS for granules from G have been defined. We extend $\|\bullet\|_{IS}$ on the family of sets $H \subseteq G$ by $\|H\|_{IS} = \{\|g\|_{IS} : g \in H\}$.

Example 4 *One can consider granules defined by sets of rules. Assume that there is a set of rules $Rule\_Set = \{(\alpha_i, \beta_i) : i = 1, \ldots, k\}$. The semantics of Rule_Set is defined by*

$$\|Rule\_Set\|_{IS} = \{\|(\alpha_i, \beta_i)\|_{IS} : i = 1, \ldots, k\}.$$

Example 5 *One can also consider as set of granules a family of all granules $(\alpha, Rule\_Set(DT_\alpha))$, where α belongs to a given subset of elementary granules.*

Example 6 *Granules defined by sets of decision rules corresponding to a given evidence are also examples of sequences of granules. Let $DT = (U, A \cup \{d\})$ be a decision table and let α be an elementary granule of $IS = (U, A)$ such that $\|\alpha\|_{IS} \neq \emptyset$. Let $Rule\_Set(DT_\alpha)$ be the set of decision rules (e.g. in minimal form) of the decision table $DT_\alpha = (\|\alpha\|_{IS}, A \cup \{d\})$ being the restriction of DT*

to objects satisfying α*. We obtain a new granule* $(\alpha, Rule\_Set\,(DT_\alpha))$ *with the semantics*

$$\|(\alpha, Rule\_Set\,(DT_\alpha))\|_{DT} = (\|\alpha\|_{IS}\,, \|Rule\_Set\,(DT_\alpha)\|_{DT}).$$

This granule describes a decision algorithm applied in the situation characterized by α*.*

Extension of granules defined by tolerance relation. We present examples of granules obtained by application of a tolerance relation.

Example 7 *One can consider extension of elementary granules defined by tolerance relation. Let* $IS = (U, A)$ *be an information system and let* τ *be a tolerance relation on elementary granules of* IS*. Any pair* (α, τ) *is called a* τ-elementary granule. *The semantics* $\|(\alpha, \tau)\|_{IS}$ *of* (α, τ) *is the family* $\{\|\beta\|_{IS} : (\beta, \alpha) \in \tau\}$.

Example 8 *Let us consider granules defined by rules of tolerance information systems. Let* $IS = (U, A)$ *be an information system and let* τ *be a tolerance relation on elementary granules of* IS*. If* ***if*** α ***then*** β *is a rule in* IS *then the semantics of a new information granule* $(\tau : \alpha, \beta)$ *is defined by* $\|(\tau : \alpha, \beta)\|_{IS} = \|(\alpha, \tau)\|_{IS} \times \|(\beta, \tau)\|_{IS}$.

Example 9 *We consider granules defined by sets of decision rules corresponding to a given evidence in tolerance decision tables. Let* $DT = (U, A \cup \{d\})$ *be a decision table and let* τ *be a tolerance on elementary granules of* $IS = (U, A)$*. Now, any granule* $(\alpha, Rule\_Set\,(DT_\alpha))$ *can be considered as a representative of information granule cluster*

$$(\tau : (\alpha, Rule\_Set\,(DT_\alpha)))$$

with the semantics

$$\|(\tau : (\alpha, Rule\_Set\,(DT_\alpha)))\|_{DT} =$$

$$\left\{\|(\beta, Rule\_Set\,(DT_\beta))\|_{DT} : (\beta, \alpha) \in \tau\right\}.$$

Dynamic granules. An elementary granule α of the information system IS is non-empty if $\|\alpha\|_{IS} \neq \emptyset$. A non-empty elementary granule β of IS is an extension of α if $\beta = \alpha \wedge \gamma$, where γ is an elementary granule. Let us consider granules defined by some subsets of

$$\{(\beta, Rule\_Set\,(DT_\beta)) : \beta \text{ is an extension of } \alpha\}.$$

The semantics of these new granules is defined as in the case of sets of granules. Any set G of granules and a granule α are specifying new granules

$$\{(\beta, Rule\_Set\,(DT_\beta)) : \beta \text{ is an extension of } \alpha \text{ and } \beta \in G\}$$

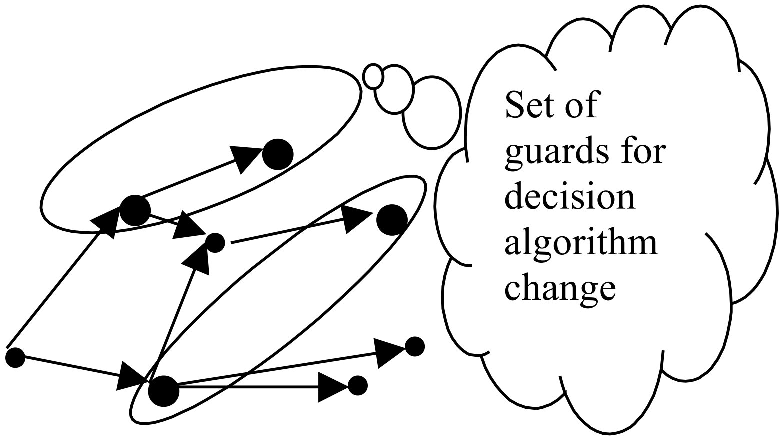

Figure 2: Two Sets of Guards

important for decision making in dynamically changing environment. A DT-path is any sequence $\pi = ((\alpha_1, R_1), \dots, (\alpha_k, R_k))$ such that α_i is an elementary non-empty granule of IS, $R_i = Rule\_Set(DT_{\alpha_i})$ for $i = 1, ..., k$ and $\alpha_i = \alpha_{i-1} \wedge \gamma_{i-1}$ for some elementary atomic granule γ_{i-1} (e.g. selector $a = v$) with an attribute $a \in A$ not appearing in α_{i-1} for $i = 2, ..., k$. A granule α_{i-1} is called a guard of π if R_{i-1} is not sufficiently close to R_i (what we denote by $non(cl_p(R_{i-1}, R_i))$, where p is the closeness degree). By $Guard(\pi)$ we denote the subsequence of $\alpha_1, \dots, \alpha_k$ consisting all guards of π. In applications it is important to search for a minimal (in cardinality) granule G satisfying the following condition: for any maximal DT-path π of extensions of α all guards β from $Guard(\pi)$ (i.e. all points in which it is sufficient to change the decision algorithm represented by the set of decision rules) are from G. Sets of guards are symbolized in Figure 2.
One can also consider dynamic granules with tolerance relation. Let $DT = (U, A \cup \{d\})$ be a decision table and let τ be a tolerance relation on elementary granules of $IS = (U, A)$. Two DT-paths $\pi = ((\alpha_1, R_1), \dots, (\alpha_k, R_k))$ and $\pi' = ((\beta_1, R'_1), \dots, (\beta_l, R'_l))$ are *τ-similar* if $(\alpha_{i_s}, \beta_{j_s}) \in \tau$ for $s = 1, \dots, r$, where $Guard(\pi) = (\alpha_{i_1}, \dots, \alpha_{i_r})$ and $Guard(\pi') = (\beta_{j_1}, \dots, \beta_{j_r})$. Let us assume τ has the following property:
if $(\beta, \alpha) \in \tau$ then the granules $Rule\_Set(DT_\alpha)$ and $Rule\_Set(DT_\beta)$ are sufficiently close.

We observe that having such tolerance relation one can search for a set G of guards of the smaller size. To specify the task is enough to change in the above formulated problem the condition for the maximal path to the following one: for any maximal path π of extensions of α there exists a τ-similar path π' to π such that all guards β from $Guard(\pi')$ (i.e. all points where it is necessary to change the decision algorithm represented by the set of decision rules) are from G.

Labeled graph granules. We discuss graph granules and labeled graph granules as notions extending previously introduced granules defined by tolerance relation and dynamic granules.

Example 10 *Let us consider granules defined by pairs* (G, E), *where* G *is a finite set of granules and* $E \subseteq G \times G$. *Let* $IS = (U, A)$ *be an information system. The semantics of a new information granule* (G, E) *is defined by* $\|(G, E)\|_{IS} = (\|G\|_{IS}, \|E\|_{IS})$, *where* $\|G\|_{IS} = \{\|g\|_{IS} : g \in G\}$ *and* $\|E\|_{IS} = \{(\|g\|, \|g'\|) : (g, g') \in E\}$.

Example 11 *Let* G *be a set of granules. Labeled graph granules over* G *are defined by* (X, E, f, h), *where* $f : X \to G$ *and* $h : E \to P(G \times G)$. *We also assume one additional condition*
if $(x, y) \in E$ *then* $(f(x), f(y)) \in h(x, y)$.

The semantics of labeled graph granule (X, E, f, h) *is defined by*

$$\{(\|f(x)\|_{IS}, \|h(x, y)\|_{IS}, \|f(y)\|_{IS}) : (x, y) \in E\}.$$

Let us summarize the above presented considerations. One can define the set of granules G as the least set containing a given set of elementary granules G_0 and closed with respect to the defined above operations of new granule construction.

We have the following examples of granule construction rules:

$$\frac{\alpha_1, \ldots, \alpha_k\text{- elementary granules}}{\{\alpha_1, \ldots, \alpha_k\}\text{- granule}}$$

$$\frac{\alpha_1, \alpha_2\text{- elementary granules}}{(\alpha_1, \alpha_2)\text{- granule}}$$

$$\frac{\alpha\text{- elementary granule}, \tau\text{- tolerance relation on elementary granules}}{(\tau : \alpha)\text{- granule}}$$

$$\frac{G\text{- a finite set of granules}, E \subseteq G \times G}{(G, E)\text{- granule}}$$

Let us observe that in case of granules constructed with application of tolerance relation we have the rule restricted to elementary granules. To obtain a more general rule like

$$\frac{\alpha\text{- graph granule}, \tau\text{- tolerance relation on graph granules}}{(\tau : \alpha)\text{- granule}}$$

it is necessary to extend the tolerance (similarity, closeness) relation on more complex objects. We discuss the problem of closeness extension in the following section.

5 Granule Inclusion and Closeness

In this section we will discuss inclusion and closeness of different information granules introduced in the previous section. Let us mention that the choice of inclusion or closeness definition depends very much on the area of application and data analyzed. This is the reason that we have decided to introduce a separate section with this more subjective part of granule semantics.

The inclusion relation between granules G, G' of degree at least p will be denoted by $\nu_p(G, G')$. Similarly, the closeness relation between granules G, G' of degree at least p will be denoted by $cl_p(G, G')$. By p we denote a vector of parameters (e.g. positive real numbers).

A general scheme for construction of hierarchical granules and their closeness can be described by the following recursive meta-rule: if granules of order $\leq k$ and their closeness have been defined then the closeness $cl_p(G, G')$ (at least in degree p) between granules G, G' of order $k+1$ can be defined by applying an appropriate operator F to closeness values of components of G, G', respectively.

A general scheme of defining more complex granule from simpler ones can be explored using rough mereological approach [4].

Inclusion and closeness of elementary granules. We have introduced the simplest case of granules in information system $IS = (U, A)$. They are defined by $EF_B(x)$, where EF_B is a conjunction of selectors of the form $a = a(x)$, $B \subseteq A$ and $x \in U$. Let $G_{IS} = \{EF_B(x) : B \subseteq A \ \& \ x \in U\}$. In the standard rough set model [3] elementary granules describe indiscernibility classes with respect to some subsets of attributes. In a more general setting see e.g. [6], [10] tolerance (similarity) classes are described.

The crisp inclusion of α in β, where $\alpha, \beta \in \{EF_B(x) : B \subseteq A \ \& \ x \in U\}$ is defined by $\|\alpha\|_{IS} \subseteq \|\beta\|_{IS}$, where $\|\alpha\|_{IS}$ and $\|\beta\|_{IS}$ are sets of objects from IS satisfying α and β, respectively. The non-crisp inclusion, known in KDD, for the case of association rules is defined by means of two thresholds t and t':

$support_{IS}(\alpha, \beta) = card(\|\alpha \wedge \beta\|_{IS}) \geq t$, and

$accuracy_{IS}(\alpha, \beta) = \frac{support_{IS}(\alpha,\beta)}{card(\|\alpha\|_{IS})} \geq t'$.

Elementary granule inclusion in a given information system IS can be defined using different schemes, e.g., by

$\nu_{t,t'}^{IS}(\alpha, \beta)$ if and only if $support_{IS}(\alpha, \beta) \geq t \ \& \ accuracy_{IS}(\alpha, \beta) \geq t'$

or

$\nu_t^{IS}(\alpha, \beta)$ if and only if $accuracy_{IS}(\alpha, \beta) \geq t$.

The closeness of granules can be defined by

$cl_{t,t'}^{IS}(\alpha, \beta)$ if and only if $\nu_{t,t'}^{IS}(\alpha, \beta)$ and $\nu_{t,t'}^{IS}(\beta, \alpha)$ hold.

Decision rules as granules. One can define inclusion and closeness of granules corresponding to rules of the form **if** α **then** β using accuracy coefficients.

Having such granules $g = (\alpha, \beta)$, $g' = (\alpha', \beta')$ one can define inclusion and closeness of g and g' by $\nu_{t,t'}(g, g')$ if and only if $\nu_{t,t'}(\alpha, \alpha')$ and $\nu_{t,t'}(\beta, \beta')$.

The closeness can be defined by

$cl_{t,t'}(g, g')$ if and only if $\nu_{t,t'}(g, g')$ and $\nu_{t,t'}(g', g)$.

Another way of defining inclusion of granules corresponding to decision rules is as follows

$\nu_t^{IS}((\alpha,\beta),(\alpha',\beta'))$ if and only if $\nu_{t_1,t_2}(\alpha,\alpha')$ and $\nu_{t_1,t_2}(\beta,\beta')$ and $t = w_1 \bullet t_1 + w_2 \bullet t_2$, where w_1, w_2 are some given weights satisfying $w_1 + w_2 = 1$ and $w_1, w_2 \geq 0$.

Extensions of elementary granules by tolerance relation. For extensions of elementary granules defined by similarity (tolerance) relation, i.e., granules of the form (α,τ), (β,τ) one can consider the following inclusion measure:

$\nu_{t,t'}^{IS}((\alpha,\tau)(\beta,\tau))$ if and only if

$\nu_{t,t'}^{IS}(\alpha',\beta')$ for any α',β' such that $(\alpha,\alpha') \in \tau$ and $(\beta,\beta') \in \tau$

and the following closeness measure:

$cl_{t,t'}^{IS}((\alpha,\tau)(\beta,\tau))$ if and only if $\nu_{t,t'}^{IS}((\alpha,\tau)(\beta,\tau))$ and $\nu_{t,t'}^{IS}((\beta,\tau)(\alpha,\tau))$.

Sets of rules. It can be important for some applications to define closeness of an elementary granule α and the granule (α,τ) . The definition reflecting an intuition that α should be a representation of (α,τ) sufficiently close to this granule is the following one:

$cl_{t,t'}^{IS}(\alpha,(\alpha,\tau))$ if and only if $cl_{t,t'}(\alpha,\beta)$ for any $(\alpha,\beta) \in \tau$.

An important problem related to association rules is that the number of such rules generated even from simple data table can be large. Hence, one should search for methods of aggregating close association rules. We suggest that this can be defined as searching for some close information granules.

Let us consider two finite sets $Rule\_Set$ and $Rule\_Set'$ of association rules defined by

$$Rule\_Set = \{(\alpha_i,\beta_i) : i = 1,\dots,k\},$$

$$Rule\_Set' = \{(\alpha_i',\beta_i') : i = 1,\dots,k'\}.$$

One can treat them as higher order information granules. These new granules $Rule\_Set$, $Rule\_Set'$ can be treated as close in a degree at least t (in IS) if and only if there exists a relation rel between sets of rules $Rule\_Set$ and $Rule\_Set'$ such that:

1. For any $Rule$ from the set $Rule\_Set$ there is $Rule'$ from $Rule\_Set'$ such that $(Rule, Rule') \in rel$ and $Rule$ is close to $Rule'$ (in IS) in degree at least t.

2. For any $Rule'$ from the set $Rule\_Set'$ there is $Rule$ from $Rule\_Set$ such that $(Rule, Rule') \in rel$ and $Rule$ is close to $Rule'$ (in IS) in degree at least t.

Another way of defining closeness of two granules G_1, G_2 represented by sets of rules can be described as follows.

Let us consider again two granules $Rule\_Set$ and $Rule\_Set'$ corresponding to two decision algorithms. By $I(\beta_i')$ we denote the set $\{j : cl_p(\beta_j',\beta_i')\}$ for any $i = 1,\dots,k'$.

Now, we assume $\nu_p\left(Rule\_Set, Rule\_Set'\right)$ if and only if for any $i \in \{1, \dots, k'\}$ there exists a set $J \subseteq \{1, \dots, k\}$ such that

$$cl_p\left(\bigvee_{j \in I(\beta'_i)} \beta'_j, \bigvee_{j \in J} \beta_j\right) \text{ and } cl_p\left(\bigvee_{j \in I(\beta'_i)} \alpha'_j, \bigvee_{j \in J} \alpha_j\right)$$

and for closeness we assume

$cl_p\left(Rule\_Set, Rule\_Set'\right)$ if and only if

$\nu_p\left(Rule\_Set, Rule\_Set'\right)$ and $\nu_p\left(Rule\_Set', Rule\_Set\right)$.

For example if the granule G_1 consists of rules: **if** α_1 **then** $d = 1$, **if** α_2 **then** $d = 1$, **if** α_3 **then** $d = 1$, **if** β_1 **then** $d = 0$, **if** β_2 **then** $d = 0$ and the granule G_2 consists of rules: **if** γ_1 **then** $d = 1$, **if** γ_2 **then** $d = 0$, then

$cl_p\left(G_1, G_2\right)$ if and only if $cl_p\left(\alpha_1 \vee \alpha_2 \vee \alpha_3, \gamma_1\right)$ and $cl_p\left(\beta_1 \vee \beta_2, \gamma_2\right)$.

One can consider a searching problem for a granule $Rule\_Set'$ of minimal size such that $Rule\_Set$ and $Rule\_Set'$ are close.

Granules defined by sets of granules. The previously discussed methods of inclusion and closeness definition can be easily adopted for the case of granules defined by sets of already defined granules. Let G, H be sets of granules.

The inclusion of G in H can be defined by

$\nu_{t,t'}^{IS}\left(G, H\right)$ if and only if for any $g \in G$ there is $h \in H$ for which $\nu_{t,t'}^{IS}\left(g, h\right)$ and the closeness by

$cl_{t,t'}^{IS}\left(G, H\right)$ if and only if $\nu_{t,t'}^{IS}\left(G, H\right)$ and $\nu_{t,t'}^{IS}\left(H, G\right)$.

Inclusion for complex granules specified by inclusion of their parts is symbolized in Figure 3.

Let G be a set of granules and let φ be a property of sets of granules from G (e.g. $\varphi\left(X\right)$ if and only if X is a tolerance class of a given tolerance $\tau \subseteq G \times G$). Then $P_\varphi\left(G\right) = \{X \subseteq G : \varphi\left(X\right) \text{ holds}\}$. Closeness of granules $X, Y \in P_\varphi\left(G\right)$ can be defined by

$cl_t\left(X, Y\right)$ if and only if $cl_t\left(g, g'\right)$ for any $g \in G$ and $g' \in H$.

We have the following examples of inclusion and closeness propagation rules:

$$\frac{\text{for any } \alpha \in G \text{ there is } \alpha' \in H \text{ such that } \nu_p(\alpha, \alpha')}{\nu_p(G, H)}$$

$$\frac{cl_p(\alpha, \alpha'), cl_p(\beta, \beta')}{cl_p((\alpha, \beta), (\alpha', \beta'))}$$

$$\frac{\text{for any } \alpha' \in \tau(\alpha) \text{ there is } \beta' \in \tau(\beta) \text{ such that } \nu_p(\alpha', \beta')}{\nu_p((\tau : \alpha), (\tau : \beta))}$$

$$\frac{cl_p(G, G') \text{ and } cl_p(E, E')}{cl_p((G, E), cl_p(G', E'))}$$

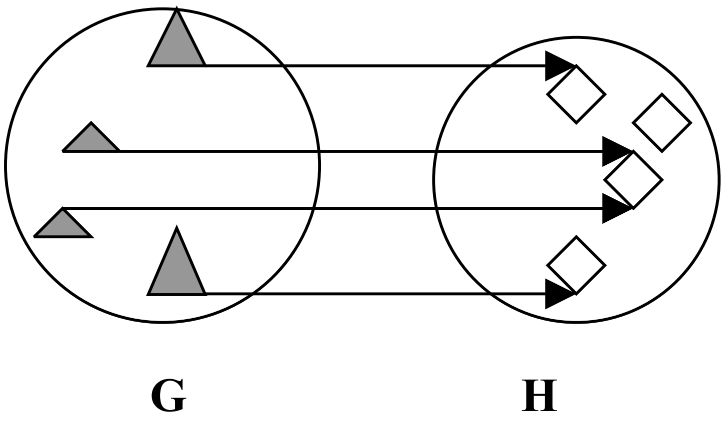

Figure 3: Two Sets of Granules

where $\alpha, \alpha', \beta, \beta'$ are elementary granules, G, G' are finite sets of elementary granules.

One can also present other discussed cases for measuring the inclusion and closeness of granules in the form of inference rules. The exemplary rules have a general form, i.e., they are true in any IS (under the chosen definition of inclusion and closeness). Some of them are derivable from others. We will see in the next part of our paper that there are also some operations of new granules construction specific for a given information system. In this case one should extract from existing data these specific inference rules.

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