Approximate Reasoning by Agents

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Abstract. We present an approach to approximate reasoning by agents in distributed environments based on calculi of information granules. Approximate reasoning schemes are basic schemes of information granule construction. An important property of such schemes is their robustness with respect to input deviations. In distributed environments, such schemes are extended to rough neural networks that transform information granules into information granules rather than vectors of real numbers into (vectors of) real numbers. Problems of learning in rough neural networks from experimental data and background knowledge are outlined.

1 Introduction

Information granulation belongs to intensively studied topics in soft computing (see, e.g., [28], [29], [30]). One of the recently emerging approaches to deal with information granulation, called granular computing (GC), is based on information granule calculi (see, e.g., [17], [24]). The development of such calculi is important for making progress in many areas like object identification by autonomous systems (see, e.g., [2], [26]), web mining (see, e.g., [6]), spatial reasoning (see, e.g., [3]) or sensor fusion (see, e.g., [1], [13]). One of the main goals of GC is to achieve computing with words (CWW) (see, e.g., [28], [29], [30]).

Any approach to information granulation should make it possible to define complex information granules (e.g., in spatial and temporal reasoning, one should be able to determine if the situation on the road is safe on the basis of sensor measurements [26] or to classify situations in complex games like soccer [25]). These complex information granules constitute a form of information fusion. Any calculus of complex information granules should make it possible to (i) deal with vagueness of information granules, (ii) develop strategies of inducing multi-layered schemes of complex granule construction, (iii) derive robust (stable) information granule construction schemes with respect to deviations of granules from which they are constructed, and (iv) develop adaptive strategies for reconstruction of induced schemes of complex information granule synthesis. To deal with vagueness, one can adopt fuzzy set theory [27] or rough set theory [12] either separately or in combination [10]. The second requirement is related to the problem of understanding of reasoning from measurements relative to perception (see, e.g., [30]) and to concept approximation learning in layered

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learning [25] as well as to fusion of information from different sources (see, e.g., [28], [29], [30]). The importance of searching for approximate reasoning schemes (*AR*-schemes, for short) as schemes of new information granule construction, is stressed in rough mereology (see, e.g., [15], [16], [16], [19]). In general, this leads to hierarchical schemes of new information granule construction. This process is closely related to ideas of co-operation, negotiations and conflict resolution in multi-agent systems [5]. Among important topics studied in relation to *AR*-schemes are methods for specifying operations on information granules; in particular, AR-schemes are useful in constructing information granules from data and background knowledge, and in supplying methods for inducing these hierarchical schemes of information granule construction. One of the possible approaches is to learn such schemes using evolutionary strategies [8]. Robustness of the scheme means that any scheme produces a higher order information granule that is a clump (e.g., a set) of close information granules rather than a single information granule. Such a clump is constructed by means of the scheme from the Cartesian product of input clumps (e.g., clusters) satisfying some constraints. The input clumps are defined by deviations (up to acceptable degrees) of input information granules from standards (prototypes).

It is worthwhile to mention that modeling complex phenomena requires us to use complex information granules representing local models (perceived by local agents) that are fused. This process involves negotiations between agents [5] to resolve contradictions and conflicts in local modeling. This kind of modeling will become more and more important in solving complex real-life problems which we are unable to model using traditional analytical approaches. If the latter approaches can be applied to modeling of such problems they lead to exact models. However, the necessary assumptions used to build them in case of complex real-life problems are often cause the resulting solutions to be *too far* from reality to be accepted as solutions of such problems.

Let us also observe, using multi-agent terminology, that local agents perform operations on information granules that are *understandable* by them. Hence, granules submitted as arguments by other agents should be approximated by means of properly tuned approximation spaces creating interfaces between agents. The process of tuning of the approximation space [23], [19] parameters in *AR*-schemes corresponds to the tuning of weights in neural networks. The methods for inducing of *AR*-schemes transforming information granules into information granules developed using rough set (see, e.g., [12], [7]) and rough mereological methods in hybridization with other soft computing approaches create a core for rough neurocomputing (RNC) (see, e.g., [11], [19]). In RNC, computations are performed on information granules.

One of the basic problems concerns relationships between information granules and words (linguistic terms) in a natural language and also a possibility to use induced *AR*-schemes as schemes matching up to a satisfactory degree reasoning schemes in natural language. Further research in this direction will create strong links between RNC and CWW.

RNC aims at defining information granules using rough sets [12], [7] and rough mereology (see, e.g., [16], [16], [19]) introduced to deal with vague concepts in hybridization with other soft computing methods like neural networks [20], fuzzy sets [10], [27], [29] and evolutionary programming [11], [8]. The methods based on the above mentioned approaches can be used for constructing of more complex information granules by means of schemes analogous to neural networks.

We outline a rough neurocomputing model as a basis for granular computing.

2 Information Granules

We assume each agent ag from a given collection Ag of agents [5] is equipped with a system of information granules $S(ag)$ specifying information granules the agent ag is perceiving and the inclusion (or closeness) relations to a degree used by ag to measure the degree of inclusion (or closenees) between information granules. A formal definition of information granule system the reader can find, e.g., in [22]. Using such system $S(ag)$ the agent ag creates its representations, e.g. in the form of decision systems [12]. The construction of information granules can be quite complex.

Let us consider classifiers as examples of information granules (see Figure 1). Classifiers are important examples of information granules because they are intensively used in machine learning and pattern recognition applications.

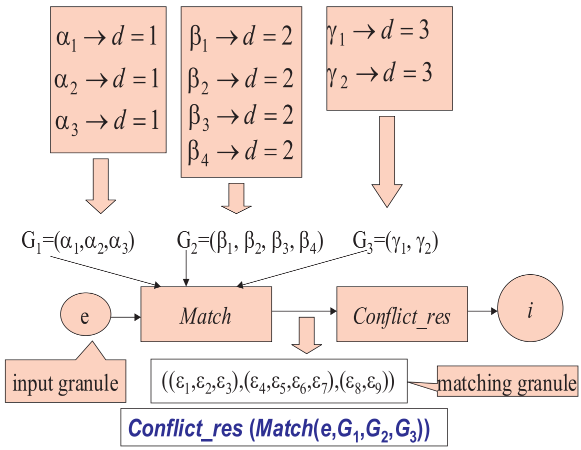

Fig. 1. An example of information granule: Classifier

First let us assume there is given a consistent decision table $DT = (U, A, d)$ with r decision values. Let E be a set of Boolean descriptors being conjunctions of elementary descriptors, i.e., expressions of the form (a, v) over attributes from $\mathcal{A} = (U, A)$ where $a \in A$ and v is a value of a.

The classifier construction from DT can be described as follows:

1. Construct granules G_j corresponding to each particular decision $j = 1, \ldots, r$ of $\mathcal{A}$ by taking a collection $\{g_{ij} : i = 1, \ldots, k_j\}$ of the left hand sides of decision rules for a given decision j for $j = 1, \cdots, r$ (in Figure 1 we have $r = 3$).
2. Construct a granule defined by a term

$$Match(e, \{G_1, \ldots, G_r\})$$

where $G_1, \ldots, G_r$ are constants and e is a variable taking values from E. Any instantiation of e defines the result of voting by all decision rules for a given object represented by e.
3. Construct a term

$$Conflict\_res(Match(e, \{G_1, \ldots, G_r\}))$$

where $Conflict\_res$ is a voting operation resolving conflicts between decision rules.

Let us observe that the decision predicted by the classifier is equal to the value of the constructed term for a particular input information granule e. We have represented a classifier construction by a term

$$Conflict\_res(Match(e, \{G_1, \ldots, G_r\}))$$

with matching and conflict resolution operations performed on information granules. Let us observe that $G_1, \ldots, G_r$ are parameters defined by the left hand sides of decision rules generated from a given decision table and e is a variable with values in E. Parameters to be tuned in such construction to induce high quality classifiers are voting strategies, matching strategies of objects against rules as well as other parameters like inclusion degrees of granules to the target granule.

3 *AR*-Schemes

AR-schemes are the basic constructs used in RNC. Such schemes can be derived from parameterized productions representing robust dependencies in data. Algorithmic methods for extracting such productions from data are discussed in [16], [21], [14]. The left hand side of each production is (in the simplest case) of the form $(st_1(ag), (\epsilon_1^{(1)}, \cdots, \epsilon_r^{(1)}), \cdots, (st_k(ag), (\epsilon_1^{(k)}, \cdots, \epsilon_r^{(k)})$ and the right hand side is of the form $(st(ag), (\epsilon_1, \cdots, \epsilon_r)$ for some positive integers k, r.

Such production represents an information about an operation o which can be performed by the agent ag. In the production k denotes the arity of operation. The operation o represented by the production is transforming standard (prototype) input information granules $st_1(ag), \cdots, st_k(ag)$ into the standard (prototype) information granule $st(ag)$. Moreover, if input information granules

$g_1, \cdots, g_k$ are close to $st_1(ag), \cdots, st_k(ag)$ to degrees $\epsilon_j^{(1)}, \cdots, \epsilon_j^{(k)}$ then the result of the operation o on information granules $g_1, \cdots, g_k$ is close to the standard $st(ag)$ to a degree at least ϵ_j where $1 \leq j \leq k$. Standard (prototype) granules can be interpreted in different ways. In particular they can correspond to concept names in natural language.

The productions described above are basic components of a reasoning system over an agent set Ag. An important property of such productions is that they are expected to be discovered from available experimental data and background knowledge. Let us also observe that the degree structure is not necessarily restricted to reals from the interval $[0, 1]$. The inclusion degrees can have a structure of complex information granules used to represent the degree of inclusion. It is worthwhile to mention that the productions can also be interpreted as a constructive description of some operations on fuzzy sets. The methods for such constructive description are based on rough sets and Boolean reasoning (see, e.g., [7], [12]).

AR-schemes can be treated as derivations obtained by using productions from different agents. The relevant derivations generating *AR*-schemes satisfy so called robustness (or stability) condition. This means that at any node of derivation the inclusion (or closeness) degree of constructed granule to the prototype (standard) granule is higher than required by the production to which the result should be sent. This makes it possible to obtain a sufficient robustness condition for whole derivations. For details the reader is referred to, e.g., [17], [18]. In cases where standards are interpreted as concept names in natural language and a reasoning scheme in natural language over the standard concepts is given, the corresponding *AR*-scheme represents a cluster of reasoning (constructions) approximately following (by means of other information granule systems) the reasoning in natural language. In the following section, we discuss in more details the concept of standard information granules.

3.1 Standards

In this section we discuss different approaches to standard information granule definition.

Standards represented by rough sets. In the simplest case, standards can be represented by the lower approximations of concepts. The degree of inclusion of a pattern supported by objects from the set $X \subseteq U$ into the lower approximation supported by the objects from the set $Y \subseteq U$ can be measured by the ratio $|X \cap Y|/|X|$ where U is the set of objects in a given decision table (representing the training sample).

However, if the lower approximation is intended to describe the concept in an extension of the training sample U, then inductive reasoning should be used to find an approximation of this lower approximation of the concept. Such approximations can be represented, e.g., by decision rules describing the lower approximation and its complement together with a method making possible to measure matching degrees of new objects and the decision rules as well as the method for conflict resolution between decision rules voting for the new objects. In such cases the degree of inclusion of any pattern in the lower approximation has more complex structure and can be represented by two vectors of inclusion degrees of this pattern in decision rules representing the lower approximation and its complement, respectively.

Using the rough set approach, one can measure not only the degree of inclusion of a concept in the lower approximation but also the degree of inclusion of a concept in other information granules defined using rough set approach such as upper approximations, boundary regions or complements of upper approximations of concepts. In this case instead of one degree one should consider a vector of degrees. However, if the lower approximation is too small the measurements based on inclusion in the standard defined by the lower approximation can be unreliable and then it can be necessary to consider other kinds of standards which can be constructed using, e.g., rough-fuzzy approach or classifier construction methods.

Standards corresponding to rough-fuzzy sets. The presented in the previous section approach can be extended to concepts defined by fuzzy sets. We will show that the dependencies between linguistic variables can be modeled by productions. Using the rough-fuzzy approach one can search for dependencies between lower approximations of differences between relevant cuts of fuzzy sets modeling linguistic variables. The productions built along such dependencies make it possible to model dependencies between linguistic variables. Moreover, the approximate reasoning on linguistic variables can be modeled by approximate reasoning schemes (AR-schemes) derived from productions.

We are going now to describe rough-fuzzy granules. We assume if X is an information granule, e.g., a set of objects, then its upper and lower approximations with respect to any subset of attributes in a given information system or decision table is an information granule too. Let us see now how such information granules can be used to define in a constructive way fuzzy concept [27] approximations.

Let $DT = (U, A, d)$ be a decision table with the decision being the restriction to the objects from U of the fuzzy membership function $\mu : U \to [0, 1]$. Consider reals $0 < c_1 < \ldots < c_k$ where $c_i \in (0, 1]$ for $i = 1, \ldots, k$. Any c_i defines c_i-cut by $X_i = \{x \in U : \mu(x) \geq c_i\}$. Assume $X_0 = U, X_{k+1} = X_{k+2} = \emptyset$.

A *rough–fuzzy granule* (*rf–granule*, for short) corresponding to $(DT, c_1, \ldots, c_k)$ is any granule $g = (g_0, \ldots, g_k)$ such that for some $B \subseteq A$

1. $Sem_B(g_i) = (\underline{B}(X_i - X_{i+1}), \overline{B}(X_i - X_{i+1}))$ for $i = 0, \ldots, k$;
2. $\overline{B}(X_i - X_{i+1}) \subseteq (X_{i-1} - X_{i+2})$ for $i = 1, \ldots, k$.

where $\underline{B}$ and $\overline{B}$ denote the B-lower and B-upper approximation operators, respectively [12] and $Sem_B(g_i)$ denotes the semantics of g_i.

Any function $\mu^* : U \to [0, 1]$ satisfying the following conditions:

1. $\mu^*(x) = 0$ for $x \in U - \overline{B}X_1$;

2. $\mu^*(x) = 1$ for $x \in \underline{B}X_k$;
3. $\mu^*(x) = c_{i-1}$ for $x \in \underline{B}(X_{i-1} - X_i)$ and $i = 2, \ldots, k-1$;
4. $c_{i-1} < \mu^*(x) < c_i$ for $x \in (\overline{B}X_i - \underline{B}X_i)$ where $i = 1, \ldots, k$, and $c_0 = 0$;

is called a B-approximation of μ.

Assume a rule **if** α **and** β **then** γ is given where α, β, γ are linguistic variables. The aim is to develop a searching method for rough-fuzzy granules g^1, g^2, g^3 approximating to satisfactory degrees α, β, γ, respectively and at the same time making it possible to discover association rules of the form **if** α' **and** β' **then** γ' with a sufficiently large support and confidence coefficients, where α', β', γ' are some components (e.g., the lower approximations of differences between cuts of fuzzy concepts corresponding to linguistic variables) of granules g^1, g^2, g^3 (modeling linguistic variables), respectively. Searching for such patterns and rules is a complex process with many parameters to be tuned. For given linguistic rules, the relevant cuts for corresponding to them fuzzy concepts should be discovered. Next, the relevant features (attributes) should be chosen. They are used to construct approximations of differences between cuts. Moreover, relevant measures should be chosen to measure the degree of inclusion of object patterns in the constructed lower approximations. One can expect that these measures are parameterized and the relevant parameters should be discovered in the process searching for productions. Certainly, in searching for relevant parameters in this complex optimization process evolutionary techniques can be used. This quality of discovered rules can be measured as a degree to which discovered rule **if** α' **and** β' **then** γ' approximates the linguistic rule **if** α **and** β **then** γ. This can be expressed by means of such parameters like degrees of inclusion of patterns α', β', γ' in α, β, γ, their supports etc.

Let us observe that for a given linguistic rule it will be necessary to find a family of rules represented by discovered patterns which together create an information granule sufficiently close to modeled linguistic rule.

One can also search for more general information granules representing clusters of discovered rules **if** α' **and** β' **then** γ' approximating the linguistic rule **if** α **and** β **then** γ. These clustered rules can be of higher quality. Certainly, this makes it necessary to discover and tune many parameters relevant for measuring similarity or closeness of rules.

The discussed problem is of a great importance in classification of situations by autonomous systems on the basis of sensor measurements [26]. Moreover, this is one of the basic problems to be investigated for hybridization of rough and fuzzy approaches.

Standards corresponding to classifiers. For classifiers we obtain another possibility. Let us consider information granules corresponding to values of terms $Match(e, \{G_1, \cdots, G_k)\}$ for $e \in E$ [21]. Any such granule defines a probability distribution on a set of possible decisions (extended by the value corresponding to *no decision predicted*). Probability for each such value is obtained simply as a ratio of all votes for the decision value determined by this information granule and the number of objects. Some probability distributions can be chosen as standards. It means, that instead of the lower approximations one can use such probability distributions. Certainly, it can be sometimes useful to choose not one such standard but a collection of them. Now, one should decide how to measure the distances between probability distributions. Using a chosen distance measure, e.g., Euclidean, it is possible to measure a degree of closeness of classified objects e, e' using the probability distributions corresponding to them. The next steps of construction of approximate reasoning rule based on classifiers is analogous to the discussed before.

One of the most interesting case is received if standards are interpreted as concepts from natural language. In this case measures of inclusion and closeness can be based on semantic similarity and closeness relations rather than on statistical properties. Constructing such measures is a challenge. This case is strongly related to the CWW paradigm. The discovered productions can to satisfactory degree be consistent with reasoning steps performed in natural language.

4 Rough Neural Networks

We extend *AR*-schemes for synthesis of complex objects (or granules) developed in [17] by adding one important component. As a result we obtain granule construction schemes that can be treated as a generalization of neural network models. The main idea is that granules sent by one agent to another are not, in general, exactly understandable by the receiving agent. This is because these agents are using different languages and usually does not exist any translation (from the sender language to the receiver language) preserving exactly semantical meaning of formulas. Hence, it is necessary to construct interfaces that will make it possible to understand received granules approximately. These interfaces can be, in the simplest case, constructed on the basis of information exchanged by agents and stored in the form of decision data tables. From such tables the approximations of concepts can be constructed using the rough set approach [24]. In general, this is a complex process because a high quality approximation of concepts can be often obtained only in dialog (involving nagotiations, conflict resolutions and cooperation) among agents. In this process the approximation can be constructed gradually when dialog is progressing. In our model we assume that for any n-ary operation $o(ag)$ of an agent ag there are approximation spaces $AS_1(o(ag), in), ..., AS_n(o(ag), in)$ which will filter (approximate) the granules received by the agent for performing the operation $o(ag)$. In turn, the granule sent by the agent after performing the operation is filtered (approximated) by the approximation space $AS(o(ag), out)$. These approximation spaces are parameterized. The parameters are used to optimize the size of neighborhoods in these spaces as well as the inclusion relation. A granule approximation quality is taken as the optimization criterion. Approximation spaces attached to any operation of ag correspond to neuron weights in neural networks whereas the operation performed by the agent ag on information granules corresponds to the operation realized on vectors of real numbers by the neuron. The generalized scheme of agents is returning a granule in response to input information granules. It can be for example a cluster of elementary granules. Hence, our schemes realize much more general computations than neural networks operating on vectors of real numbers.

We call extended schemes for complex object construction *rough neural networks* (for complex object construction). The problem of deriving such schemes is closely related to perception (see, e.g., [30]). The stability of such networks corresponds to the resistance to noise of classical neural networks.

Let us observe that in our approach the deductive systems are substituted by productions systems of agents linked by approximation spaces, communication strategies and mechanism of derivation of AR-schemes. This revision of classical logical notions seems to be important for solving complex problems in distributed environments.

5 Decomposition of Information Granules

Information granule decomposition methods are important components of methods for inducing of AR-schemes from data and background knowledge. Such methods are used to extract from data, local decomposition schemes called produtions [18]. The AR-schemes are constructed by means of productions. The decomposition methods are based on searching for the parts of information granules that can be used to construct relevant higher level patterns matching up to a satisfactory degree the target granule.

One can distinguish two kinds of parts (represented, e.g., by sub-formulas or sub-terms) of AR-schemes. Parts of the first type are represented by expressions from a language, called the *domestic* language L_d, that has known semantics (consider, for example, semantics defined in a given information system [12]). Parts of the second type of AR-scheme are from a language, called *foreign* language L_f (e.g., natural language), that has semantics definable only in an approximate way (e.g., by means of patterns extracted using rough, fuzzy, rough–fuzzy or other approaches). For example, the parts of the second kind of scheme can be interpreted as soft properties of sensor measurements [2].

For a given expression e, representing a given scheme that consists of subexpressions from L_f first it is necessary to search for relevant approximations in L_d of the foreign parts from L_f and next to derive global patterns from the whole expression after replacing the foreign parts by their approximations. This can be a multilevel process, i.e., we are facing problems of discovered pattern propagation through several domestic-foreign layers.

Productions from which AR-schemes are built can be induced from data and background knowledge by pattern extraction strategies. Let us consider some of such strategies. The first one makes it possible to search for relevant approximations of parts using the rough set approach. This means that each part from L_f can be replaced by its lower or upper approximation with respect to a set B of attributes. The approximation is constructed on the basis of relevant data table [12], [7]. With the second strategy parts from L_f are partitioned into a number of sub-parts corresponding to cuts (or the set theoretical differences between cuts) of fuzzy sets representing vague concepts and each sub-part is approximated by means of rough set methods. The third strategy is based on searching for patterns sufficiently included in foreign parts. In all cases, the extracted approximations replace foreign parts in the scheme and candidates for global patterns are derived from the scheme obtained after the replacement. Searching for relevant global patterns is a complex task because many parameters should be tuned, e.g., the set of relevant features used in approximation, relevant approximation operators, the number and distribution of objects from the universe of objects among different cuts and so on. One can use evolutionary techniques [8] in searching for (semi-) optimal patterns in the decomposition.

It has been shown that decomposition strategies can be based on rough set methods for decision rule generation and Boolean reasoning [16], [9], [24]. In particular, methods for decomposition based on background knowledge can be developed. The interested reader is referred to [21], [14].

Conclusions. We have discussed a methodology for synthesis of AR-schemes and rough neural networks. For more details the reader is referred to [16], [17], [19], [23], [24]. The reported research topics are very much related to multi-agent systems. We would like to emphasize two of them, namely:

1. *Algorithmic methods for synthesis of AR-schemes.* It was observed (see, e.g., [19]) that problems of negotiations and conflict resolutions are of great importance for synthesis of AR-schemes. The problem arises, e.g., when we are searching in a given set of agents for a granule sufficiently included or close to a given one. These agents, often working with different systems of information granules, can derive different granules and their fusion will be necessary to obtain the relevant output granule. In the fusion process, the negotiations and conflict resolutions are necessary. Much more work should be done in this direction by using the existing results on negotiations and conflict resolution. In particular, Boolean reasoning methods seem to be promising for solving such problems. Another problem is related to the size of production sets. These sets can be of large size and it is important to develop learning methods for extracting *small* candidate production sets in the process of extension of temporary derivations out of huge production sets. For solving this kind of problems, methods for clustering of productions should be developed to reduce the size of production sets. Moreover, dialog and cooperation strategies between agents can help to reduce the search space in the process of AR-scheme construction from productions.
2. *Algorithmic methods for learning in rough neural networks.* A basic problem in rough neural networks is related to selecting relevant approximation spaces and to parameter tuning. One can also look up to what extent the existing methods for classical neural methods can be used for learning in rough neural networks. However, it seems that new approach and methods for learning of rough neural networks should be developed to deal with real-life applications. In particular, it is due to the fact that high quality approximations of concepts can be often obtained only through dialog and

negotiations processes among agents in which gradually the concept approximation is constructed. Hence, for rough neural networks learning methods based on dialog, negotiations and conflict resolutions should be developed. In some cases, one can use directly rough set and Boolean reasoning methods (see, e.g., [24]). However, more advanced cases need new methods. In particular, hybrid methods based on rough and fuzzy approaches can bring new results [10].

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