# Constructing Rough Mereological Granules of Classifying Rules and Classifying Algorithms

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**Abstract**

Rough Set Theory cf. [3] was conceived as an approach toward analysis of uncertainty as well as incompleteness. Its basic assumptions going back to logical and philosophical analysis by – among others – Leibniz, Frege and Russell – is that objects perceived by a given set of *attributes* should be regarded as *indiscernible* whenever the attributes have same values on them (*Leibnizian identity*). Sets of objects which may be represented as unions of classes of the indiscernibility relation are then complete (*exact, certain)* while all other sets may be described by means of approximations with complete sets. The framework of rough sets allows for construction of classifying as well as decision rules and algorithms cf. [9] as well as for many applications to real life problems (*op. cit.*).

Rough Mereology cf. [6], [7], [8], [11] is a paradigm based on the predicate of *being a part to a degree* and as such falls in the province of *mereological theories of reasoning* based on the notion of a *part* which go back to the tradition of the Polish School in particular to the work of S. Leśniewski cf. [2]. Rough Mereology is a paradigm allowing for a synthesis of main ideas of two potent paradigms for reasoning under uncertainty : Fuzzy Set Theory and Rough Set Theory. We present applications of Rough Mereology to the important theoretical idea put forth by Lotfi Zadeh [12], [13] i.e. Granularity of Knowledge. Granules of Knowledge are constructed in the framework of Rough Mereology via its *class operator* which allows for aggregation of objects close enough (or, *similar in a satisfactory degree*) with respect to the rough inclusion operator (which measures the degree of being a part for pairs of objects). This allows for constructing Logics for Reasoning in Multi–Agent environment. We present a basic outline of this approach. We propose a formal language for encoding reasoning schemes (the *Synthesis Grammar*) and here we carry the idea of Synthesis Grammar to a higher level of abstraction by constructing Granules of classifying rules as well as classifying algorithms. We finally discuss briefly the analogy between rough mereological and neural computations leading to the idea of hybrid rough–neural computation schemes.

**Keywords**

knowledge discovery, rough sets, rough mereology, information granulation, classification algorithms and rules

## 1 Introduction

We begin with basic notions of Rough Set Theory [3], [6], [7], [8], [9]. Knowledge is represented in rough set approach by means of an information system $\mathbf{A} = (U, A)$ where $U$ is a (current) set of objects, $A$ is a (current) set of (conditional) attributes and each attribute $a \in A$ is a mapping on the set $U$ i.e. $a : U \rightarrow V_a$ where $V_a$ is the set of values of $a$.

Objects with identical descriptions are not discernible: for each $u \in U$ and a set $B \subseteq A$ of attributes, we define the *information set* $Inf_B(u)$ of $u$ over the set$B$ by

(INF) $Inf_B(u) = \{(a, a(u)) : a \in B\}$

and we express indiscernibility of objects with respect to $B$ by the relation $IND_B$ of $B$*–indiscernibility*:

(IND) $IND_B(u, w) \Longleftrightarrow Inf_B(u) = Inf_B(w)$.

The indiscernibility relation $IND_B$ partitions the set $U$ into classes $[u]_B$; these classes are regarded by us as *elementary granules* of knowledge and they in turn may be represented in predicate calculus of descriptors *(attribute=value)*: a descriptor $(a = v)$ is satisfied by $u$ if $a(u) = v$ and given $[u]_B$ with $B = \{a_{i_1}, a_{i_2}, .., a_{i_k}\}$ and $a_{i_j}(u) = v_{i_j}$, we find that the formula $\wedge_j(a_{i_j} = v_{i_j})$ is satisfied by and only by elements of $[u]_B$. In what follows we refer to $[u]_B$ also as to the $(B, v)$*–template* where $v =< v_{i_j} >$.

Dependencies between templates say $(B, v)$ and $(C, w)$ are expressed by formulae of the form

$\wedge_j(a_{i_j} = v_{i_j}) \Longrightarrow \wedge_k(b_{i_k} = w_{i_k})$

where $B = \{a_{i_j}\}$, $C = \{b_{i_k}\}$; these dependencies in turn may be regarded as *classifying rules* allowing to define $C$–values of an object in terms of its $B$–values and they may be represented as *pairs* $([u]_B, [u']_C)$ of elementary granules with appropriate $u, u'$.

The power (quality) of a classifying rule $R : ([u]_B, [u']_C)$ is characterized by two parameters viz. (cf. [1] for a discussion)

$\alpha(R) = \frac{|[u]_B \cap [u']_C|}{|[u]_B|}$ (*classification accuracy*);

$\kappa(R) = \frac{|[u]_B \cap [u']_C|}{|[u']_C|}$ (*coverage*).

These parameters express what part of the first granule is contained in the second and vice versa (clearly one may give these parameters also a probabilistic interpretation as unbiased estimators of the corresponding probabilities cf. [5]).

Both accuracy and coverage are based on the rough membership function [4]; in recent applications the need has been stressed for more relaxed approach based not only on indiscernibility but also on a variety of tolerance (similarity) relations [9]. The need arises thus for tools for expressing similarity in information systems and for algorithms based on these tools.

Here, we propose to introduce rough mereological approach to the granulation problem in which $IND$-classes are replaced with mereological classes of satisfactorily close objects and granules. We discuss in the following Ontology, Mereology and Rough Mereology in Information Systems.

## 2 Ontology in Information Systems

Our ontology is adapted from Ontology of S. Leśniewski [2]. We modify it in notational aspect. Ontology is a theory of the copula "is" and we render it here as the membership $\in$ to be read *is*. We adopt the Ontology Axiom of Leśniewski.

$$X \in Y \Longleftrightarrow \exists Z. Z \in X$$
$$\wedge \forall U, W.(U \in X \wedge W \in X \Longrightarrow U = W)$$
$$\wedge \forall T.(T \in X \Longrightarrow T \in Y).$$

These three conjuncts express respectively that: (i) $X$ is a non–empty name; (ii) $X$ is a singleton (i.e. an individual); (iii) any entity called $X$ is also called $Y$.

This defines the meaning of the copula $\in$. In particular, $X \in X$ states that $X$ is an individual.

### 2.1 Examples: Rough Set Ontology in Information Systems

As individuals, we consider elementary granules. For an elementary granule (template) $(C, v)$ we denote by the symbol $[C, v]$ its *meaning*: $[C, v] = \{u \in U : \forall a \in C. a(u) = v_a\}$. We will write down the granule $(C, v)$ as the pair $((C, v), [C, v])$. We introduce operations on elementary granules denoted by $\oplus, \odot, \ominus$

1. $((C, v), [(C, v)]) \oplus ((D, w), [(D, w)])$ is the granule whose meaning consists of those objects which are either in $[(C, v)]$ or in $[(D, w)]$;
2. $((C, v), [(C, v)]) \odot ((D, w), [(D, w)])$ is the granule whose meaning consists of those objects which fall both in $[(C, v)]$ and $[(D, w)]$;
3. $\ominus((D, w), [(D, w)])$ where is the granule whose meaning consists of those objects which fall in $U$ but not in $[(D, w)]$.

We will often denote a granule $((C, v), [(C, v)])$ with the symbol $g$ and then $[g]$ will denote the meaning of $g$. Then, a general granule $g$ may be written down in a canonical form as $\oplus_{i=1}^{k}((C_i, v_i), [C_i, v_i])$ with $[g] = \bigcup_{i=1}^{k}[C_i, v_i]$ where each $((C_i, v_i), [C_i, v_i])$ is an elementary granule.

## 3 Mereology in Information Systems

Our Mereology is an adaptation of Mereology proposed by Leśniewski [2] which has offered a formal treatment of the predicate of being a part. We begin with the notion of a *part* functor (*pt*, for short). *pt* is introduced into Ontology by means of additional axioms. We formalize $pt(Y)$ as a set-forming functor of entities that are parts of $Y$. $pt(Y)$ is defined only for individual $Y$.

(ME1) $X \in pt(Y) \wedge Y \in pt(Z) \Longrightarrow Xpt(Z)$; (*symmetricity of pt*).

(ME2) $non(Xpt(X))$. (*non–reflexivity of pt*).

The concept of an improper part is reflected in the notion of an *element el* defined as follows: $X \in el(Y) \Longleftrightarrow X \in pt(Y) \vee X = Y$.

Basic feature of Mereology of Leśniewski is the presence of the class functor $Kl$ making properties into single objects and defined as follows.

$$X \in Kl(Y) \Longleftrightarrow$$

$$\forall Z.(Z \in Y \Longrightarrow Z \in el(X))$$

$$\wedge \forall Z.(Z \in el(X) \Longrightarrow \exists U, W.$$

$$U \in Y \wedge W \in el(U) \wedge W \in el(Z)).$$

These respective conditions state that each individual in $Y$ is an element of $Kl(Y)$ and the class $Kl(Y)$ consists of all individuals which have an element in common with an individual in $Y$.

One also requires

(ME3) $X \in Kl(Y) \wedge Z \in Kl(Y) \Longrightarrow Z = X$; (*$Kl(Y)$ is an individual*).

(ME4) $\exists Z.Z \in Y \Longleftrightarrow \exists Z.Z \in Kl(Y)$ (*the class existence*).

The class operator will be used by us as a granule–forming tool in the sequel.

## 4 Rough Mereology

Rough Mereology [6], [7], [11] has been proposed and studied as a tool for approximate reasoning. Its primitive notion is that of a *rough inclusion* i.e. a functor $\mu(r)$ of being a part in degree at least $r$ for each $r \in [0, 1]$.

The following is a list of basic postulates about Rough Mereology. We introduce a graded family $\mu_r$, where $r \in [0, 1]$ is a real number from the unit interval, of functors which would satisfy the following requirements ($\mu_r(X)$ is a new property derived from $X$ via $\mu_r$ and we use relational notation $X \in \mu_r(Y)$ for the statement : $X$ is a part of $Y$ in degree at least $r$):

(RM1) $X \in \mu_1(Y) \Longleftrightarrow X \in el(Y)$; (*a part in degree 1 is an element*).

(RM2) $X \in \mu_1(Y) \Longrightarrow \forall Z.(Z \in \mu_r(X) \Longrightarrow Z \in \mu_r(Y))$; (*monotonicity*).

(RM3) $X = Y \wedge X \in \mu_r(Z) \Longrightarrow Y \in \mu_r(Z)$; (*identity is a $\mu$–congruence*).

(RM4) $X \in \mu_r(Y) \wedge s \leq r \Longrightarrow X \in \mu_s(Y)$; (*degree at leat r*).

### 4.1 Rough inclusions in information systems

The following procedure defines a rough inclusion in an information system $(U, A)$.

**Procedure 1**

1. Consider a partition $P = \{A_1, A_2, ..., A_k\}$ of $A$.
2. Select a family of coefficients: $W = \{w_1, w_2, ..., w_k\}$ where $w_i \geq 0$, any $i$, and $\sum_{i=1}^{k} w_i = 1$.
3. Define $IND(A_i)(x, y) = \{a \in A_i : a(x) = a(y)\}$.
4. Let $r = \sum_{i=1}^{k} w_i \frac{card(IND(A_i)(x,y))}{card(A_i)}$.
5. Declare $x \in \mu_r(y)$.

$\mu_r$ thus defined is a pre–rough inclusion as it is defined on objects; we now propose a method for extending a measure defined for elements of two sets to a measure on these two sets.

Assume that we are given two individuals $X, Y$ being classes of (finite) names: $X = Kl(X')$, $Y = Kl(Y')$ and that we have defined values of $\mu$ for pairs $T, Z$ of individuals where $T \varepsilon X'$, $Z \varepsilon Y'$.

We extend $\mu$ to a measure $\mu^*$ on $X, Y$ by letting:

$$Y \varepsilon \mu_r^*(X) \text{ for } r = min_{Z \in Y'}\{max_{T \in X'} max\{s : Z \varepsilon \mu_s(T)\}\}.$$

It may be proved straightforwardly that

**Proposition 1.** *The measure $\mu^*$ satisfies (RM1)–(RM4).*

**Rough inclusions based on frequency count.** In this case our strategy is based on counting frequencies by means of the rough membership function applied to specifically defined counted objects; particular strategies depend on the type of individual objects we consider. We point to few cases.

1. In case our individual objects $g, g'$ are $B$–elementary granules, we may apply the strategy of counting the number of $B$–indiscernibility classes in respectively $[g] \cap [g']$ and $[g']$. Accordingly, for $[g] = \bigcup_{i=1}^{k}[(B, v_i)]$ and $[g'] = \bigcup_{j=1}^{m}[(B, w_j)]$, we let $g' \varepsilon \mu_r(g)$ where
$$r = \frac{\mid \{[(B, v_i)] : i \leq k\} \cap \{[(B, w_j)] : j \leq m\} \mid}{m};$$
2. In case our individual objects $g, g'$ are elementary granules in $\mathsf{A}= (U, A)$, we may apply the strategy of counting rows: for two elementary granules $g, g'$ with $[g] = \bigcup_{i=1}^{k}[(B_i, v_i)]$ and $[g'] = \bigcup_{j=1}^{m}[(C_j, w_j)]$, we let
$$g \varepsilon \mu_r(g')$$
where
$$r = \frac{\mid [g] \cap [g'] \mid}{\mid [g'] \mid}.$$

We also may apply the strategy of counting indiscernibility classes assuming

$$g\varepsilon\mu_r(g')$$

where

$$r = \frac{\mid \{[(B_i, v_i)] : i \leq k\} \cap \{[(C_j, w_j)] : j \leq m\} \mid}{m}.$$

3. We may apply a hybrid approach counting rows for indiscernibility classes and extending the received closeness measure to general individuals. First then, we define $\mu$ on atomic elementary granules $g = ((B, v), [(B, v)])$, $g' = ((C, w), [(C, w)])$: we begin with the set $IND(g, g') = \{a \in A : a \in B \cap C \wedge v(a) = w(a)\}$ and then we let $g'\varepsilon\mu_r(g)$ where $r = \frac{|IND(g,g')|}{|B|}$. Thus, the degree of partial containment of $g'$ in $g$ is determined by frequency count of identical elementary descriptors in templates $(B, v)$ and $(C, w)$.
Now, given individual entities being elementary granules $g, g'$ with $[g] = \bigcup_{i=1}^{k}[(B_i, v_i)]$ and $[g'] = \bigcup_{j=1}^{m}[(C_j, w_j)]$, we let

$$g\varepsilon\mu_r^*(g')$$

where

$$r = min_{[(C_j,w_j)]}\{max_{[(B_i,v_i)]}max\{s : [(C_j, w_j)]\varepsilon\mu_s([(B_i, v_i)])\}\}.$$

*Example 1.* We give a simple example concerning the last method of calculating the measure $\mu$. We begin with an example of an information system presented in Table 1.

| | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $u_1$ | 1 | 0 | 1 |
| $u_2$ | 1 | 0 | 0 |
| $u_3$ | 1 | 1 | 0 |
| $u_4$ | 0 | 1 | 1 |
| $u_5$ | 0 | 1 | 0 |
| $u_6$ | 1 | 0 | 1 |
| $u_7$ | 1 | 1 | 0 |

**Table 1.** *Binary1*: An example of an information table

Consider $B = \{a_1, a_2\}$, $C = \{a_2, a_3\}$ , $v =< 1, 0 >$, $w =< 0, 1 >$ . For $g = ((B, v), [B, v])$, $g' = ((C, w), [C, w])$, we have $IND(g, g') = \{a_2\}$ and accordingly, $g'\varepsilon\mu_{0.5}^*(g)$.

### 4.2 Rough mereological component of granulation

The functors $\mu_r$ may enter our discussion of a granule:

1. Concerning the definitions of accuracy and coverage we may replace in them the rough membership function $\mu$ with a function $\mu_r$ possibly better suited to a given context.

2. The process of clustering may be described in terms of the class functor of mereology.

We will realize 1,2 in the sequel. Our setting will be the distributed environment of a multi–agent system.

## 5 Adaptive Calculi of Granules in Distributed Systems

We construct a mechanism for transferring granules of knowledge among agents by means of transfer functions induced by rough mereological connectives extracted from their respective information systems [7].

We now recall basic ingredients of our scheme of agents [6], [7], [11].

### 5.1 Distributed systems of agents

We refer to a model for approximate synthesis in a distributed system proposed in [6]– [8], [9].

Consider a distributed (multi-agent) system $M_A$= $\{Ag, Link, Inv\}$ where $Ag$ is a set of agents, $Link$ is a finite list of words over $Ag$ and $Inv$ is a set of inventory objects. Each $t$ in $Link$ is a word $ag_1ag_2...ag_kag$ meaning that $ag$ is the parent node and $ag_1ag_2...ag_k$ are children nodes in an elementary team $t$; both parties are related by means of the operation $o_t$ which makes from a tuple $(x_1, ..., x_k)$ of objects, resp. at $ag_1, .., ag_k$ the object $o_t(x_1, .., x_k)$ at $ag$. Leaf agents $Leaf$ are those $ag$ which are not any parent node. They operate on objects from $Inv$.

In addition, each agent $ag$ is equipped with an information system $\mathbf{A(ag)}$= $(U(ag), A(ag)$ and a rough inclusion $\mu_{ag}$ on $U(ag)$ (cf. Procedure 1); a set $St(ag) \subseteq U(ag)$ of *standard individuals* is also defined for any $ag$.

Reasoning in $M_A$ goes by way of standards and rough inclusions $\mu_{ag}$ at any $ag$. Instrumental in this reasoning process are rough connectives $f_{\sigma,t}$ where $\sigma$ $=(st_1, ..., st_k, st)$ is a set of standards such that $o_t(st_1, .., st_k) = st$ (we call $\sigma$ *admissible*). They propagate rough inclusion values from children nodes to the parent node according to the formula

$$\forall i. x_i \in \mu_{ag_i,r_i}(st_i) \Longrightarrow o_t(x_1, .., x_k) \in \mu_{ag,f(r_1,...,r_k)}(st).$$

**Approximate logic of synthesis** We assume for simplicity that the distributed system $M_A$ consists of $Ag$=$\{ag_1, ag_2, ag\}$ with $Link$=$ag_1ag_2ag$ and the operation $o$. This will not restrict the universality of our discussion but it will simplify the notation and make it easier to understand essential features of this approach.

We introduce a simplified logic $L(Ag)$ [7], [11] in which we can express global properties of the synthesis process.
Elementary formulae of $L(Ag)$ are $< st(a), \varepsilon(a) >$ where $st(a) \in St(a), \varepsilon(a) \in [0,1]$ for any $a \in Ag$. Formulae of $L(ag)$ form the smallest extension of the set of elementary formulae closed under propositional connectives $\vee$, $\wedge$, $\neg$ and under the modal operators $\Box$, $\Diamond$.
For $x \in U(a)$, we say that $x$ satifies $< st(a), \varepsilon(a) >$, in symbols:

$$x \vdash < st(a), \varepsilon(a) >,$$

iff $x \in \mu(a)_{\varepsilon(a)}(st(a))$.

Notice that $st(a)$ may choose a formula (a choice is by no means unique) in descriptor language which it does satisfy and $x$ has to be close enough to $st(a)$ in order to satisfy the chosen formula in degree $\varepsilon(a)$.

We extend satisfaction over formulae by recursion as usual.

By a *selection* over $Ag$ we mean a function $sel$ which assigns to each agent $a$ an object $sel(a) \in U(a)$. For two selections $sel, sel'$ we say that *sel induces sel'*, in symbols $sel \rightarrow_{Ag} sel'$ when $sel(a) = sel'(a)$ for $a = ag_1, ag_2$ and $sel'(ag) = o(ag)(sel'(ag_1), sel'(ag_2))$.

We extend the satisfiability predicate $\vdash$ to selections: for an elementary formula $< st(a), \varepsilon(a) >$, we let $sel \vdash < st(a), \varepsilon(a) >$ iff $sel(a) \vdash < st(a), \varepsilon(a) >$.

We now let $sel \vdash \Diamond < st(a), \varepsilon(a) >$ when there exists a selection $sel'$ satisfying the conditions: $sel \rightarrow_{Ag} sel'; sel' \vdash < st(a), \varepsilon(a) >$.

In terms of $L(Ag)$ it is possible to express the problem of synthesis of an approximate solution to the problem posed to $Ag$.

In the process of top - down communication, a requirement $\Psi$ received by the scheme from an external source (which may be called a *customer*) is decomposed into approximate specifications of the form $< st(a), \varepsilon(a) >$ for any agent $a$ of the scheme. The decomposition process is initiated at the agent $ag$ and propagated along the scheme. We now are able to formulate the synthesis problem.

**Synthesis problem**

*Given* $\alpha : < st(ag)), \varepsilon(ag) >$ *find a selection sel with the property* $sel \vdash \alpha$.

A solution to the synthesis problem with a given formula $\alpha$ is found by negotiations among the agents based on uncertainty rules and their successful result can be expressed by a top-down recursion as follows: it is sufficient that each agent $ag_i$ choose a standard $st(ag_i) \in U(ag_i)$ and a coefficient $\varepsilon(ag_i) \in [0,1]$ such that

**Criterion 5.1**

1. $\sigma = (st(ag_1), st(ag_2), st(ag))$ is admissible i.e. $o(st(ag_1), st(ag_2)) = st(ag)$;
2. $f_\sigma(\varepsilon(ag_1), \varepsilon(ag_2)) \geq \varepsilon(ag)$.

We call an $\alpha$ - *scheme* an assignment of a formula $\alpha(a) : < st(a), \varepsilon(a) >$ to each $a \in Ag$ in such manner that 1, 2 in Crit. 5.1 are satisfied and $\alpha(ag))$ is $\alpha$. We denote this scheme with the symbol $sch(\alpha)$.

We say that a selection $sel$ is *compatible* with a scheme $sch(\alpha)$, in case $sel(a) \in \mu(a)_{\varepsilon(a)}(st(a))$ for $a = ag_1, ag_2$.

The goal of negotiations can be summarized now as follows.

**Proposition 5.1**

Given a formula $\alpha : < st(ag)), \varepsilon(ag)) >$, **if** a selection $sel$ is compatible with a scheme $sch(\alpha)$ **then** $sel \vdash \Diamond\alpha$.

## 6 Calculi of Elementary Granules

We construct in a given system $M_A$ of agents for each agent $a$, granules by means of a rough inclusion $\mu_a$ of the agent $a$.

For a standard $st(a)$ and $\varepsilon(a)$, we denote by the symbol $gr(st(a), \varepsilon(a))$ (*the granule of size* $\varepsilon(a)$ *about* $st(a)$) the class $Kl_{\varepsilon(a)}(st(a))$ where $Kl_{\varepsilon(a)}(st(a))$ is the class (i.e. the set) of those $x$ for which $x \in \mu_{a,\varepsilon(a)}(st(a))$.

**Example 1**

Let us define the rough inclusion $\mu_{a,r}$ according to Procedure 1 with $w_1 = 1$ i.e. $x \in \mu(a)_r(y) \iff r \geq \frac{|IND(x,y)|}{|A_a|}$.

For a given template $(B, v)$, the elementary granule $gr(B, v)$ defined by $(B, v)$ consists of all $x$ with $a_i(x) = v_i$ for $a_i \in B$.

We regard now $(B, v)$ as a standard say $st(a)_{B,v}$; for this standard, we define the rough inclusion $\mu_{a,st(a)_{B,v},r}$ according to Procedure 1 with $w_1 = 1$ i.e. $x \in \mu_{a,st(a)_{B,v},r}(y) \iff r \geq \frac{|\{a \in A_a - B : a(x) = a(y)\}|}{|A_a|}$. Then, given $\varepsilon$, the granule $gr(st(a)_{B,v}, \varepsilon)$ consists of those $x$ which agree with $st(a)_{B,v})$ on $B$ and additionally agree with this standard on at least $\varepsilon \times 100$ percent of the remaining attributes.

Thus, granules $gr(st(a)_{B,v}, \varepsilon)$ provide a covering of the elementary granule $gr(B, v)$ by similarity classes of $\mu_{a,st(a)_{B,v}}$.

### 6.1 Synthesis in Terms of Granules

We say that $gr(st(a), \varepsilon(a))$ satisfies a formula $\alpha :< st(a), \varepsilon'(a) >$,

$$gr(st(a), \varepsilon(a)) \vdash \alpha$$

in case $\varepsilon(a) \geq \varepsilon'(a)$.

Given admissible $\sigma = \{st(ag), st(ag_1), st(ag_2)\}$ and $\varepsilon(ag), \varepsilon(ag_1), \varepsilon(ag_2)$ with $f_\sigma(\varepsilon(ag_1), \varepsilon(ag_2)) \geq \varepsilon(ag)$ (i.e. 1,2 in Crit. 5.1 are satisfied) we observe that: **if** $x \in gr(st(ag_1), \varepsilon(ag_1))$, $y \in gr(st(ag_2), \varepsilon(ag_2))$, **then** $o(x, y) \in gr(st(ag), \varepsilon(ag))$.

We may state the sufficiency of synthesis condition in terms of granules as follows.

**Proposition 6.1**

For the formula $\alpha :< st(a), \varepsilon(a) >$ :

**if** $x \in gr(st(ag_1), \varepsilon(ag_1))$, $y \in gr(st(ag_2), \varepsilon(ag_2))$ with 1, 2 in Crit. 5.1 satisfied **then** $o(x, y) \vdash \alpha$.

It suffices thus that for a given granule $gr(st(ag), \varepsilon(ag)$, agents $ag_1$, $ag_2$ send to $ag$ granules, respectively, $gr(st(ag_1), \varepsilon(ag_1))$ and $gr(st(ag_2), \varepsilon(ag_2))$.

## 7 Associated Synthesis Grammars

The above may be formulated in terms of a grammar $\Gamma$ and a language $L(\Gamma)$ whose words code sufficient synthesis conditions [8]. With each agent $a \in Ag$, we associate a grammar $\Gamma(a) = (N(a), T(a), P(a))$. To this end, we assume that a finite set $\Xi(a) \subset [0,1]$ is selected for each $a$. We let $N(a) = \{(s_{st(a)}, t_{\varepsilon(a)}) : st(a) \in St(a), \varepsilon(a) \in \Xi(a)\}$ where $s_{st(a)}$ is a non–terminal symbol corresponding in a one - to - one way to the standard $st(a)$ and similarly $t_{\varepsilon(a)}$ corresponds to $\varepsilon(a)$.

The set of terminal symbols $T(ag)$ is defined for $ag$ by letting
$T(ag) = \cup_{i=1,2}\{\{(s_{st(ag_i)}, t_{\varepsilon(ag_i)}) : \varepsilon(ag_i) \in \Xi(ag_i)\} : i = 1, 2\}$.
The set $P(ag)$ contains productions of the form
$(s_{st(ag)}, t_{\varepsilon(ag)}) \longrightarrow (s_{st(ag_1)}, t_{\varepsilon(ag_1)})(s_{st(ag_2)}, t_{\varepsilon(ag_2)})$
where $st(ag_1), st(ag_2), st(ag)$ , $\varepsilon(ag), \varepsilon(ag_1), \varepsilon(ag_2)$ satisfy 1, 2 in Crit. 5.1.

We define a grammar system $\Gamma = (T(ag), (\Gamma(a) : a = ag \vee a = Input), S)$ by introducing an additional agent $Input$ with the non - terminal symbol $S$, terminal symbols of $Input$ being non-terminal symbols of $ag$ and productions of $Input$ of the form:
$S \Longrightarrow (s_{st(ag))}, t_{\varepsilon((ag))})$

The meaning of $S$ is that it codes an approximate specification (requirement) for an object; productions of $Input$ code specifications for approximate solutions in the language of the agent $ag$. Subsequent rewritings produce terminal strings of the form
$(s_{st(ag_1)}, t_{\varepsilon(ag_1)})(s_{st(ag_2)}, t_{\varepsilon(ag_2)})$.
We have

**Proposition 7.1**

Suppose $(s_{st(ag_1)}, t_{\varepsilon(ag_1)})(s_{st(ag_2)}, t_{\varepsilon(ag_2)})$ is obtained from

$$S \longrightarrow (s_{st(ag))}, t_{\varepsilon(ag))})$$

by subsequent rewriting by means of productions in $\Gamma$. Then given any selection $sel$ with $sel(ag_i) \in \mu(ag_i)_{\varepsilon(ag_i)}(st(ag_i))$ for $i = 1, 2$ we have

$$sel \models \Diamond < st(ag), \varepsilon(ag) > .$$

**Remarks**

1. Synthesis grammars constructed above reflect processes in multi-agent systems which arise in a multi-agent system involved in cooperation, negotiation and conflict-resolving actions when attempting to provide a solution to a specification of a problem posed to its root.

2. Complexities of membership problems for languages generated by synthesis grammars may be taken ex definitione as complexities of the underlying synthesis processes.

## 8 Synthesis of Classifying Rules and Classifying Algorithms

As mentioned above classifying rules may be represented as pairs of granules. Thus, it is the first task to establish means for measuring the rough mereological distance between granules.

Consider granules $g, g'$ in the universe $U$ endowed with a rough inclusion $\mu$. We propose the following *min-max* formula:

$$g \in \mu_r^*(g') \Longleftrightarrow min_x max\{r : x \in \mu_r(y) : y \in g'\} \geq r.$$

Then

**Proposition 8.1**

$\mu^g$ is a rough inclusion on granules as it satisfies (RM1)–(RM4).

**Example 2**

Consider templates $(B, v)$ and $(C, w)$ in an information system $\mathbf{A} = (U, A)$ with the rough inclusion $\mu$ defined in Procedure 1 with $w_1 = 1$.

It may be found straightforwardly that for $g = gr(B, v)$, $g' = gr(C, w)$, we have the worst–case estimate

$$g \in \mu_r^g(g') \Longleftrightarrow r \geq 1 - \frac{k-s}{n}$$

where $k = |C|$, $s$–the number of indices $j$ with $j \in B \cap C$, $v_j = w_j$.

We have thus a trade–off: a short template (i.e. $k$ small) means closeness of distinct granules and large size of the granule $g'$.

We extend this measure to pairs of granules i.e. clasifying rules by letting:

$$g \in \mu_r^g(g') \wedge g_1 \in \mu_s^g(g_1') \Longrightarrow$$
$$(g, g_1) \in \mu_{\top(r,s)}^{g*}(g', g_1')$$

where $\top$ is a *t-norm*.

Then $\mu^{g*}$ is a rough inclusion on classifying rules.

### 8.1 Rough mereological connectives on classifying rules

Given our system $M_A$, rough mereological connectives $f$ may be extended to connectives propagating uncertainty about classifying rules. Although it would be possible to derive a general formula from our results above, we rather give an illuminating example.

**Example 3: fusion of data**

Consider $M_A$ with $ag_1, ag_2, ag$ where information systems of agents are $A_{ag_1}=(U, A_1)$, $A_{ag_2}=(U, A_2)$ and $A_{ag}=(U, A_1 \cup A_2)$ i.e. $ag$ fuses tables of $ag_1$ and $ag_2$.

We find a formula for $F$, the rough mereological connective for granules generated by templates.

Consider thus templates $(B_1, v_1)$, $(C_1, w_1)$ at $ag_1$, $(B_2, v_2)$, $(C_2, w_2)$ at $ag_2$ and $(B = B_1 \cup B_2, v = v_1 \cup v_2)$ and $(C = C_1 \cup C_2, w = w_1 \cup w_2)$ at $ag$ along with the corrresponding elementary granules for $\mu$ as in Example 2.

We have by Example 2 that :

1. $gr(B_1, v_1) \in \mu^g_{r_1}(gr(C_1, w_1))$ with $r_1 \leq 1 - \frac{k_1 - s_1}{n_1}$;
2. $gr(B_2, v_2) \in \mu^g_{r_2}(gr(c_2, w_2))$ with $r_2 \leq 1 - \frac{k_2 - s_2}{n_2}$;
3. $gr(B, v) \in \mu^*_r(gr(C, w))$ with $r \leq 1 - \frac{(k_1 - s_1) + (k_2 - s_2)}{n_1 + n_2}$.

From 1.–3. it follows that the connective $F$ satisfies the formula

$$F(1 - \varepsilon_1, 1 - \varepsilon_2) \geq 1 - max(\varepsilon_1, \varepsilon_2) = min(1 - \varepsilon_1, 1 - \varepsilon_2).$$

**An Application: a classifier query decomposition**
In the setting of Example 3, we may select $(C, w)$. Then the query

$$? < (., .), (C, w), \delta >$$

means that we are searching for $(B, v)$ with $gr(B, v) \in \mu^g_\delta(gr(C, w))$ i.e. for a classifier with a sufficient quality (measured by $\delta$).

Then, the problem may be decomposed: it suffices that $ag_1, ag_2$ find granules $gr(B_1, v_1)$, $gr(B_2, v_2)$, respectively, satisfying:

- $gr(B_1, v_1) \in \mu^g_{1-\varepsilon_1}(gr(C_1, w_1))$;
- $gr(B_2, v_2) \in \mu^g_{1-\varepsilon_2}(gr(C_2, w_2))$

with $max(\varepsilon_1, \varepsilon_2) \leq 1 - \delta$.
The results presented here allow to

1. formulate counterparts of granule calculi for granules of classifiers;
2. define synthesis grammars in terms of classifiers;
3. extend these results to classification algorithms as these are finite collecctions of classifiers so one more application of our results would yield appropriate formulae in case of classification algorithms.

## 9 A Neural Model of Rough Mereological Computation

There is a parallelism between the proposed above calculi of granules in distributed systems and neural computing. Let us point to some analogies cf. [10].

1. Any elementary team of agents $t = ag$ may be regarded as a model of a neuron with inputs $ag_1, ag_2, ..., ag_k$, the output $ag$, and a parameterized family of activation functions represented as rough connectives $f_{\sigma,t}$ where $\sigma = (st_1, ..., st_k, st)$ is an admissible set of standards
2. Values of rough inclusions in the formula

$$\forall i. x_i \varepsilon \mu_{r_i, ag_i}(st_i) \Longrightarrow o_t(x_1, .., x_k) \varepsilon \mu_{f(r_1, ..., r_k), ag}(st)$$

are counterparts of weights in a traditional neural network. Let us observe that in our case the resulting network is a parameterized system of simple networks, indexed by synthesis schemes ($\alpha$–schemes)

3. Learning in this new kind of a neural network is based also on back-propagation mechanisms in which the incoming signal (a specification $\Psi$) is assigned a proper $\alpha$–scheme and a proper set of weights set in negotiation and cooperation processes among local teams and agents therein

These processes of learning would require new algorithms and one possible way out here is to base the process of learning on familiar techniques of neural networks by encoding all the constructs in a neural network whose activation functions are tractable (e.g. piece–wise differentiable) approximations to rough mereological connectives. As a result, we would obtain a closed–loop system providing feedback information from the distributed system to the neural network. The theory and practice of such systems is to come in future.

## 10 Conclusions

The reader of this work, after a scrutiny of the content, has undoubtedly noticed and made themselves familiar with the main ingredients of this approach: the formalism based on mereological ideas, the construction of reasoning schemes, the granule–forming mechanism, the analysis of correctness of problem solving procedure along with additional aspects i.e. the linguistic counterpart given in the guise of synthesis grammars and languages and rough–neuro–computing scheme.

In this way we propose to realize programs of *granulation of knowledge* and *computation with granules*. At the same time we propose a way toward *computing with words*: interfaces between natural language corpora and synthesis languages will allow for performing computing with words *sensu stricto*.

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## References

1. J. Bazan, Hung Son Nguyen, Sinh Hoa Nguyen, P. Synak, and J. Wróblewski, 2000, Rough set algorithms in Classification Problem, in; L. Polkowski, S. Tsumoto, and T. Y. Lin (eds.), *Rough Set Methods and Applications. New Developments in Knowledge Discovery in Information Systems*, Physica Verlag/ Springer Verlag, Heidelberg, pp. 49–88.
2. S. Leśniewski, 1927, O podstawach matematyki, *Przegląd Filozoficzny*, 30, pp.164-206; 31, pp. 261-291; 32, pp. 60-101; 33, 77-105; 34, 142-170; cf. also: S. Leśniewski, 1982, On the foundations of mathematics, Topoi 2, pp. 7–52.

3. Z. Pawlak, 1992, *Rough Sets: Theoretical Aspects of Reasoning about Data*, Kluwer, Dordrecht.
4. Z. Pawlak and A. Skowron, 1994, Rough membership functions, in: R.R. Yaeger, M. Fedrizzi, J. Kacprzyk (eds.), *Advances in the Dempster Shafer Theory of Evidence*, John Wiley and S., New York, pp. 251-271.
5. Z. Pawlak, 1999, Decision rules, Bayes' rule and rough sets, in: Ning Zhong, A. Skowron, S. Ohsuga (eds.), new Directions in Rough sets, Data Mining and Granular-Soft Computing, LNAI vol. 1711, Springer Verlag, Berlin, pp. 1–9.
6. L. Polkowski and A. Skowron, 1994, Rough mereology, in: *Lecture Notes in Artificial Intelligence*, vol. 869, Springer Verlag, Berlin, pp. 85-94.
7. L. Polkowski and A. Skowron, 1996, Rough mereology: a new paradigm for approximate reasoning, *International J. Approximate Reasoning*, 15(4), pp. 333-365.
8. L. Polkowski and A. Skowron, 1998, Grammar systems for distributed synthesis of approximate solutions extracted from experience, in: Gh. Paun and A. Salomaa (eds.), *Grammatical Models of Multi-Agent Systems*, Gordon and Breach Sci. Publ., Amsterdam, pp. 316-333.
9. L. Polkowski and A. Skowron (eds.), 1998, *Rough Sets in Knowledge Discovery*, vols. 18, 19 in the Series: *Studies in Fuzziness and Soft Computing*, J. Kacprzyk (ed.), Physica Verlag (Springer Verlag).
10. L. Polkowski and A. Skowron, 2001, Rough mereological calculi of granules: Rough Set approach to Computation, *Computational Intelligence*, in print.
11. A. Skowron and L. Polkowski, Rough mereological foundations for design, analysis, synthesis and control in distributed systems, *Information Sciences. An Intern. J.*, 104(1-2) (1998), pp. 129-156.
12. L.A. Zadeh, 1996, Fuzzy logic = computing with words, *IEEE Trans. on Fuzzy Systems*, 4, pp. 103-111.
13. L. A. Zadeh, 1997, Toward a theory of fuzzy information granulation and its certainty in human reasoning and fuzzy logic, *Fuzzy Sets and Systems*, 90, pp. 111-127.