E.6. Fuzzy Sets and Rough Sets in Data Mining

Witold Pedrycz
Department of Electrical & Computer Engineering
University of Alberta
Edmonton, AB T6G 2G6 Canada
ph. (403) 492.4661
fax. (403) 492.1811
e-mail: pedrycz@ee.ualberta.ca
and
Systems Research Institute, Polish Academy of Sciences
Warsaw, Poland

Andrzej Skowron
Institute of Mathematics
Warsaw University
Banacha 2, 02-097 Warsaw, Poland
ph. +48 22 6583449
fax. +48 22 6583448
e-mail: skowron@mimuw.edu.pl

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Abstract

We discuss various relationships of fuzzy sets and rough sets with KDD. We describe the main advances of rough set and fuzzy set methods in solving KDD problems, point out research directions in fuzzy sets and rough sets stimulated by KDD as well as characterize potential impact of these methodologies on KDD.

Introductory comments

Fuzzy set and rough set constructs presented in Sections B6 and B7 are essential for a general conceptual and algorithmic setting in data mining. However, it is apparent that fuzzy sets and rough sets arise as realizations of a far more general and fundamental concepts of information granulation and granular computing. We outline relationships between granular computing and KDD. This will help to present the relationships of fuzzy sets and rough sets with KDD. Information granulation (and granular computing afterwards) is a process of constructing information granules and their further processing. Information granules are conceptual entities embracing collections of detailed data, say numbers into a form of a single entity - information granule. The elements are pulled together owing to their functional similarity, similarity (proximity) or some other criterion of likeness [Za2], [Za4]. Processing of information granules is more general, does not concentrate on details and captures a way in which information granules are transformed. The concept of granules is appealing. A way in which IT is realized at the formal end is not unique: depending on application, we may use set theory (where information granules are modeled as sets), fuzzy set theory (in case of fuzzy sets), rough set theory (for rough sets). When information granules are expressed as probability density functions, then we exploit the framework of probability theory. We reveal relationships between granular computing and data mining showing that there is an ongoing mutually beneficial interaction and impact between these two. In particular, we show how fuzzy sets and rough sets provide a conceptual and algorithmic environment for data mining. At the same time, we highlight how data mining impacts the research agenda of these two environments.

In general, as illustrated in Figure 1, we may consider a general two-phase scheme of data mining. The first phase is devoted to granulation of information: here we identify or express some essential chunks of information that are deemed essential for knowledge representation and data summarization. Afterwards, in the second phase we employ one of the formal platforms of granular computing (rough sets, fuzzy sets, rough-fuzzy sets etc.) using which we develop associations between such granules, carry out classification tasks, communicate results to an end-user, etc. It is very likely that an interaction with the user may call for some modifications or further enhancements; these are realized via a feedback link as again indicated in Figure 1.

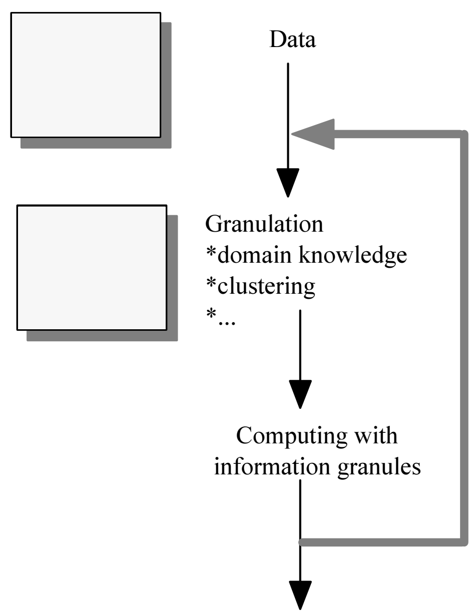

Figure 1. Two-phase model of data mining involving information granulation and granular computing

The above scheme fully complies with one of the key objectives of all data mining activities that is user-friendliness manifesting in the granular form of discovered patterns. Obviously, various relationships (associations) data mined and revealed at the level of information granules are compact and easy to understand by the user. The language of associations is more comprehensive than the language of plain formulas (for instance, regression lines) developed at the level of numeric data. Information granules hide all detailed and inessential relationships as not being pertinent to the understanding the essence of the data. The level of specificity can be easily affected by changing the level of granularity, that is using information granules that are less or more detailed and consisting of the respective number of elements. Both fuzzy sets and rough sets support a construction of information at different levels of granules. They also provide with a formal way of quantifying the notion of granularity.

E6.1. Fuzzy Sets and KDD

Fuzzy sets, as discussed in Section B7, are granular constructs that promote representation and processing of concepts with gradual boundaries. The continuity and gradual changes in membership values are essential features of many pursuits of human beings dealing with classification or decision-making problems [Za4], [Pe4].

The role of fuzzy sets in KDD

A number of key pursuits support the use of fuzzy sets as an essential technology of KDD.

In a nutshell, the role of fuzzy sets is to make KDD systems more user-friendly and user-oriented. This becomes crucial as the relevance of the fuzzy set based architectures depends very much upon an efficient interaction with the user/designer. By their nature, the role of fuzzy sets can substantial enhancement of the front-end and back-end interfaces of KDD systems. In the first case, the user can define a vocabulary of linguistic terms (modeled via fuzzy sets and fuzzy relations) that are afterwards exploited as the generic conceptual entities - linguistic landmarks throughout the ensuing algorithms of revealing the web of associations in databases. Recall that each fuzzy set or fuzzy relation (in the multidimensional case) are models of information granules in the setting of fuzzy sets. Interestingly, the structure (that is granularity), reflected often by size, and a distribution of information granules imply the form of associations to be discovered. These information granules help establish relevance and interestingness of the associations. In this way, the user and/or designer assume more proactive role in the data mining activities. The user can query the data mining system through its interface by modifying the size of some information granules. When building back-end interface, fuzzy sets help interpret the results. Rather than relying on a single number (being the outcome of some DM activities and describing various notions such as confidence level, correlation coefficients, etc.), fuzzy sets help quantify these in terms of a collection of elements with varying degrees of membership. As a consequence, the results are more descriptive and easy to interpret and visualize. They may also raise awareness as to the quality of data mining by signaling the user the need of further analysis or advise him to proceed with more caution with the interpretation of the presented results. These two interfaces realized with the aid of fuzzy sets are in line with the philosophy of visualization for KDD. Fuzzy sets are easy to define to reflect the intuitive meaning of the information granules exploited by the user. The formal apparatus is well established so that these two interfaces can be easily designed and modified.

In some cases, information granules - fuzzy sets need to be constructed from numeric data. Here fuzzy clustering is an important endeavor leading to the construction of information granules - fuzzy sets or fuzzy relations. The objective function based clustering algorithms form one of the essential classes of the grouping [KR], [HKR], [DK]. An interesting generalization of the clustering comes in the form of partial supervision [Pe3] or context-based clustering [Pe2]. These scenarios are aimed at capturing some "hints" coming from the user/designer of the data mining system (say, some directions as to the structure to be looked for). Clustering with partial supervision is positioned in-between algorithms of supervised learning and unsupervised learning.

The technology of fuzzy sets is useful in the design of rule based systems. Fuzzy sets help avoid a severe britleness problem that is inherently associated with any rule-based representation. The rule

- if input is A then output is B

"fires" (becomes activated or relevant) if a given fact X is embraced by A. Now if A is modeled as a set, then if X is included in A, the output (conclusion) is equal to B. Note, however, that if A may have a variety of interpretations (say, A denotes a concept of *high* temperature, *low* inflation rate, *safe* speed, etc. and the rule-based system hinges on set theory) then depending upon the interpretation of A that may be subjective and vary from person to person, the conclusion is either B or not. In this sense, the use of the set-theoretic machinery in this framework associates with the brittleness effect. Fuzzy sets help alleviate this problem: as A is a fuzzy set, any X "activates" A to a certain degree. This, in turn, implies the activation of B to some degree. The same process of gradual rule firing occurs for all rules in the rule-based system which, all in all, eliminates the effect of a binary switching between the rules.

Fuzzy sets augment some well-known algorithms of machine learning such as decision trees [Mi][Qu]. Here, the role of fuzzy sets manifests in several ways

- through the formation of the attribute space whose elements are modeled as fuzzy sets or fuzzy relations [Ja1, Ja2, HMB, ISN, UOH]. Decision trees require attributes with finite values (that is they have to discretized) and this discretization becomes accomplished via fuzzy sets

- by providing with the vehicle of propagating levels of matching of the available data with the attributes of the decision tree [Ja1]. As attributes - fuzzy sets can be activated to a certain extent, a number of nodes of the decision trees are involved in the propagation of the activation levels and finally a number of final nodes of the tree. A detailed mechanism of computing how the activation levels are propagated down the tree is developed based on the techniques of fuzzy sets

Influence of KDD on Fuzzy Sets

There is a visible impact of KDD (and its key thrusts) onto the research agenda of fuzzy sets. Several clear points to be made deal with the following:

- Complexity analysis and complexity reduction of fuzzy set constructs Interestingly, fuzzy sets as dealing with gradual membership grades are quite complex and demanding at the processing side. KDD makes this point clear: huge databases require a lot of processing. The processing that is excessively time consuming, no matter how useful and general it could be, may not acceptable. This situation calls for a thorough review of the basic constructs of fuzzy sets and in-depth analysis of various alternatives (such as e.g., different models of fuzzy set operations with an emphasis on those that save at the computational side).

- The same complexity analysis may require some approximation (and simplification) of fuzzy sets, say those arising in the form of sets and rough sets.

- The well-advanced, fully developed knowledge representation schemes of KDD as well as efficient KDD schemes can be a starting point for further generalizations pursued in the realm of fuzzy sets.

E6.2. Rough Sets and KDD

In recent years we have witnessed a rapid, worldwide growth of interest in rough set theory and its applications (see e.g. [Pa1], [Sl1], [Ll3], [Or1], [PSb], [Pal], [Ts1]). The theory has been followed by the development of software systems that implement rough set methods. Rough sets are applied in domains so diverse as medicine, finance, telecommunication, vibration analysis, conflict resolution, intelligent agents, pattern recognition, control theory, signal analysis, process industry, marketing, etc.

The role of rough sets in KDD

A family of approximation spaces defines the search space for data models in rough set approach. Any approximation space in this family is distinguished by some parameters. Searching strategies for optimal (sub-optimal) parameters are basic rough set tools in searching for data models and knowledge. There are two main types of parameters. The first ones are used to define object sets, called neighborhoods, the second are measuring the inclusion or closeness of neighborhoods. We are going to describe them informally (formal definition of parameterized approximation space can be found, e.g, in [PaS3]). The basic assumption of the classical rough set approach [Pa1], shared with other approaches like machine learning, pattern recognition or statistics, is that objects are perceived by means of some features (e.g. formulas being the results of measurement of the form *attribute=value* called *descriptors* [Pa1].). Hence, some objects can be indiscernible (indistinguishable) or similar to each other. The sets of indiscernible or similar objects expressible by some formulas are called *neighborhoods*. In the simplest case the family of all neighborhoods create a partition of the universe. In more general case it defines a covering. Formulas defining the neighborhoods are basic building blocks from which the approximate descriptions of other sets (decision classes or concepts) are induced. Usually, like in machine learning, the specification of concepts is incomplete, e.g., given by examples and counterexamples. Having incomplete specification of concepts, one can induce only approximate description of concepts by means of formulas defining the neighborhoods. Hence it follows that it will be useful to have parameterized formulas (e.g. in the simplest case $a>p$ & $b<q$ where a,b are attributes and p, q are parameters) so that by tuning their parameters one can select formulas being relevant for inducing concept approximation. A formula is relevant for concept description if it defines a large neighborhood still included to a sufficient degree in approximated concept.

In the simplest case the formulas defining neighborhoods are conjunctions of *descriptors*. Parameters to be tuned can be of different sort like the number of conjunction connectives in the formula or the interval boundaries in case of discretization of real value attributes. In more general case, these formulas can express the results of measurement or perception of observed objects and represent complex information granules. Among such granules can be decision algorithms labeled by feature value vectors (describing an actual situation in which algorithm should be performed), clusters of such granules defined by their similarity or hierarchical structures of such granules (see e.g. [SST]). These complex granules become more and more important for qualitative reasoning, in particular for spatial reasoning [RS1].

Extraction of relevant partitions (coverings) defined by neighborhoods

The process of concept approximation in rough set framework can be described as searching for partitions (coverings) coarser than defined by actual neighborhoods, relevant to inducing the concept approximations of high quality. There are two main characteristic steps of rough set methods. First is based on computing of appropriate basic constructs of rough sets, i.e., reducts and the second on tuning them, by computing reduct approximations, to receive the best neighborhoods. The reducts are constructed using Boolean reasoning (see Section B6) and object discernibility. The quality of concept approximation can be measured using machine learning techniques but can be also measured taking into account other measures than used in KDD, such as covering sufficiently large part of concept by small number of strong patterns.

Searching for concept approximations is related to feature extraction and feature selection well known in pattern recognition or machine learning (see Section B6). Moreover, it is also crucial to knowledge discovery [Fay], in particular to scientific discovery [V-P], [ShL], [Lan]. For example, scientific discovery [V-P] is using, as a main source of power, relatively general knowledge, including knowledge to search combinatorial spaces. Hence, it is important to discover efficient searching strategies. This includes the processes of inducing the relevant features and functions over which these strategies are constructed as well as the structure of searching strategy induced from such constructs. The goal of discovery [ShL], [Lan] is to find knowledge that is novel, plausible and understandable. Certainly, these soft concepts should be induced up to a sufficient degree, i.e., their approximations should be induced to specify the main constraints in searching for knowledge. In this sense the concept approximation is the basic step not only for machine learning or for pattern extraction but also for knowledge discovery and scientific discovery. Certainly, in the latter cases the inducing processes of concept approximations are much more complex and searching for such approximations creates a challenge for researchers.

Tuning the inclusion degree between sets of objects

From the point of view of KDD it is important to consider two relations on formulas, namely *inclusion* and *closeness*. For KDD instead of classical crisp inclusion or equivalence it is more appropriate to consider *inclusion in a degree* and *closeness in a degree*. Several approaches has been developed basing on this idea (see e.g. [PS0], [Zyt], [AMS], [Za1]). A typical example of such inclusion appears in case of association rules [AMS]. Another example is related to extracting the *default rules* from data (for details see [Mo1], [NSS]). Using rough set approach different kinds of reduct approximations allow for a degree of *impurity* in preserving partition, covering or set inclusion (see e.g. [SN1] and Section B6). Assuming a fixed partition (covering) of objects, the set approximations are induced by tuning of parameters specifying the degree of set inclusion. In this way data models are extracted from data using rough set approach.

Tuning set approximations by tuning inclusion degree

The classical rough set approach [Pa1] relies on crisp inclusion (see Section B6). For applications in KDD more relaxed, than the discussed classical case, approaches have been developed. Instead of crisp (exact) inclusion one can use inclusion in a degree and define the set approximations on its basis [ZI1], [PS1], [ST1]. This helps to solve various KDD tasks like searching for dependencies in a degree instead of exact ones or for simple, understandable by the users and interesting approximate concept description instead of their complex exact descriptions.

Quality measures

Optimization of approximation space parameters is based on searching for relevant inclusion degrees and relevant partitions (coverings) in this sense that, e.g., the concept description based on corresponding approximations performs better while classifying new objects.

Extracting relevant parameters is usually based on optimization of some measures of the concept approximation quality on a given set of objects. The quality measure should be chosen in such a way that once the optimal (or sub-optimal) description of concept has been extracted with respect to the chosen quality measure, then the induced description turns out to be also valid for unseen so far objects. Let us note that the constructs generated by the rough set methods are controlled using statistical testing procedures (see Section B6). The examples of typical measures are based on the size of the boundary regions, positive region, entropy of these regions or on the *minimal description length principle* [RI1], [DG1], [SN1], [PSb].

The last principle, often used in machine learning, has been also adopted by rough set approach. There are some specific steps offered by rough set approach, which we now shortly discuss. The extraction of shorter descriptions of concepts is realized by searching for the most general neighborhoods still relevant for concept approximation, i.e., included in a sufficient degree in approximated concept. In rough set approach this process is realized in a special way, namely by searching for constructs called *reducts* and their approximations. It can be characterized as extraction of coarser partitions (or coverings) of the object universe still relevant for concept description (for details see Section B6).

It has been shown [SN1], [PSb] that the process of searching for different kinds of reducts can be realized by Boolean Reasoning procedures (see Section B6). The combination of rough set approach with Boolean Reasoning [Br] (see also Section B6) creates a unified methodology for specifying and extracting efficiently different kinds of reducts and their approximations which are used as basic constructs of concept approximations and more general knowledge discovery (for more details see Section B6).

Generated knowledge, in particular data models, are usually sub-optimal with respect to the minimal description length principle. This is because of the high computational complexity of problems searching for the optimal models. Moreover, models extracted by using the minimal length principle usually have to be tuned (because of incomplete or/and noisy data) to obtain solutions of satisfactory quality. Typical methods are based on tuning entropy [DG1] or on sampling (see e.g. [Ba1]).

Rough sets offer sound and solid theoretical foundations in the process of inducing concept descriptions. The developed methodology based on rough sets and Boolean reasoning provided tools for construction of efficient heuristics for problem solving. At the same time it allowed to understand the common core and computational complexity of all these problems: necessity of efficient derivation of short implicants of large Boolean functions (see Section B6 and [SKA]).

Influence of KDD on rough sets

We would like to mention some of new research directions in rough sets stimulated by research in KDD.

New approaches have been developed to deal with large data bases [ImM]. For example, one of the technique is based on the decomposition (into binary trees) of large data tables using so called *templates* (see e.g. [NSS]). Templates are descriptions of regular sub-domains of the universe of objects, e.g., large groups of bank customers having sufficiently many common features. They are labeling non-leaf nodes of decomposition binary trees. Any leaf of decomposition trees is labeled by subtables of objects satisfying conditions on the path from the root to that leaf. The data are decomposed until the tables labeling leaves have the feasible size with respect to the methods for decision rules generation. Another method for discretization of real value features of large relational databases [GGR] is presented in [Ng1]. The method is based on some statistical information of discretized data and allows to find a semi-optimal *cut* (partition) of the discretized range by using only O(logn) SQL queries, where n is the number of records.

The rough set methods are reported as a useful front end for neural networks (for references see the papers and bibliography in [Pal], [PSb], [SzM]). In particular, for reducing the number of input variables.

The need to develop data mining methods from data tables with hierarchical attributes and complex values of attributes, e.g., representing algorithms or plans stimulated research on developing rough set methods for solving problems related to such data. One can consider here prediction of biological models changing on the basis of molecular DNA data [Sch] or prediction of plan changing by autonomous system [RS1].

Several other interesting research projects based on rough sets are recently reported like learning from data of conflict resolutions between different classifiers [SzM] or developing visualization interfaces for rough set methods including, e.g., *geometry of reducts* [PSb]. Some other research directions on rough sets are mentioned in other sections of the handbook related to rough sets (see Section B6).

We have mentioned above that the goal of knowledge discovery can be identified with searching for strategies constructing complex information granules representing approximations of soft-concepts like novel, interesting, plausible and understandable in considered domains. Let us also observe that qualitative process representation, qualitative reasoning, spatial reasoning, perception and measurement instruments, collaboration and communication, embodied agents are only some topics of research directions mentioned in [ShL] as important for scientific reasoning and discovery. The mentioned above topics are very much in the scope of computing with words and granular computing [Za2-3]. Rough set extension called rough mereology ([PS0-1], [PSa]) has been proposed as a tool for approximate reasoning to deal with such problems [SST], [LP]. Schemes of reasoning in rough mereology approximating soft patterns seem to be crucial for making progress in knowledge discovery [PS2]. In particular this approach has been used to build a calculus on information granules

[PSa], [SST] as a foundation for computing with words [Za2-3]. Among the discussed issues related to KDD there are generalized soft association rules, synthesis of interfaces between sources exchanging concepts and using different languages, problems in spatial reasoning [RS1] and data mining in Internet. Further interactions of rough mereology and KDD will certainly bring new results for both areas.

E6.3 Hybridization of Fuzzy Sets and Rough Sets

In the previous sections, we have identified the roles played by fuzzy sets and rough sets in KDD. It has become apparent that there is a need for some hybridization to make processes of information granulation more efficient in a rapport with the computing requirements posed by real-world data. Furthermore, one may think of building hybrid constructs that capture the essence of fuzzy sets and rough sets [DP3], [DP4], [PaS]. Rough - fuzzy hybridization methods give tools for KDD.

Fuzzification of rough concepts

The classical rough set approach is based on crisp sets. Reducts, basic constructs of rough sets, define crisp sets used to concept definition. Often making these sets more soft by their fuzzification allows achieving a higher quality of concept approximations. Examples of such constructs are: fuzzy cuts and hyperplanes [NS2], [MSz] or fuzzification of decision rules generated from corresponding reduts [NH1].

Let us also mention that set approximations based on fuzzy indiscernibility relation (e.g. defined by fuzzy similarity relation [GMS]) have shown to be useful for concept approximation.

Further investigations of techniques transforming rough concepts into fuzzy ones will certainly show more interesting results.

Approximation of fuzzy concepts by rough concepts

Rough set methods can be used to define fuzzy concepts approximately. In this case first one can look for relevant α-cuts of the fuzzy set and next approximate them with respect to known condition features [Ya1]. Problem of choosing relevant cuts is analogous to the problem of relevant feature extraction. From computational complexity point of view it is a hard problem and can be solved approximately by discovery of learning strategies. One can observe that the relevant cuts should be "well" approximated (i.e. new objects with high chance should be properly classified to sets defined by them) as well as they should give together "good" approximation of the target fuzzy set. This approach can be used for constructive definition of fuzzy membership function by rough set approach. Its importance for KDD can be illustrated by data summarization in natural language (i.e. using soft concepts) or flexible query answering [PVK].

Hybrid models of information granules, information granulation and granular data mining

We have pointed out above how models of information granules and granulation processes enrich the processes and models of data mining. The need for the studies of the hybrid models of information granules arises when we are faced with an issue of interoperability between various tasks or subsystems of data mining that could be realized in various frameworks of granular computing. This is a place for hybridization of rough and fuzzy approaches. In particular, rough-fuzzy hybrid systems (see e.g. [Pal]) have great perspectives for further applications in KDD to extract approximations of soft patterns in complex environment.

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References

[AMS] Agrawal, R., Mannila, H., Srikant, R., Toivonen, H., Verkano, A.: Fast Discovery of Association Rules, Fayyad U.M., Piatetsky-Shapiro G., Smyth P., Uthurusamy R. (Eds.): Advances in Knowledge Discovery and Data Mining, The AAAI Press/The MIT Press (1996) 307-328

[Abi] Anthony, M., Biggs, N.: Computational Learning Theory, Cambridge Tracts in Theoretical Computer Science 30, Cambridge University Press, Cambridge, 1991

[Ba1] Bazan J.G. : A comparison of dynamic and non-dynamic rough set methods for extracting laws from decision system, in: Polkowski, L., Skowron, A. (eds.): Rough Sets in Knowledge Discovery **1**: Methodology and Applications, Physica-Verlag, Heidelberg (1998) 321-365

[Br] Brown, F.M.: Boolean Reasoning, Kluwer Academic Publishers, Dordrecht, 1990

[Ch] Chacrabarti, S., Dom, B. E., Kumar, S.R., Raghavan, P., Rajagopalan, S., Tomkins, A., Gibson, D., Kleinberg, J.: Mining the web's link structure, Computer, August 1999, 60-67

[DK] Dave, R. N., Krishnapuram, R.: Robust clustering methods: a unified view, IEEE Trans. on Fuzzy Systems **5** (1997) 643-662

[DP1] Dubois, D., Prade, H.: Fuzzy sets and systems: Theory and applications. Academic Press, 1980

[DP2] Dubois, D., Prade, H.: Twofold fuzzy sets and rough sets - Some issues in knowledge representation. Fuzzy Sets and Systems **23** (1987) 3-18

[DP3] Dubois, D., Prade, H.: Rough fuzzy sets and fuzzy rough sets. International J. General Systems **17** (1990) 191-209

[DP4] Dubois, D., Prade, H.: Putting Fuzzy Sets and Rough Sets together. In: [Sl1], 203-232

[DG1] Duentsch, I., Gediga, G. (2000): Rough set data analysis. In: Encyclopedia of Computer Science and Technology, Marcel Dekker (to appear)

[Fay] Fayyad, U.M., Piatetsky-Shapiro, G., Smyth, P., Uthurusamy, R. (Eds.): Advances in Knowledge Discovery and Data Mining, The AAAI Press/The MIT Press, 1996

[GMS] Greco, S., Matarazzo, B., Słowiński, R.: Rough set theory to decision analysis. In: R. Felix (ed.), Proceedings of the Third European Workshop on Fuzzy Decision Analysis and Neural Networks for Management, Planning and Optimization, June 16-19, 1998, Dortmund, Germany (1998), 1-28

[GGR] Ganti, V., Gehrke, J., Ramakrishnan, R.: Mining very large databases. Computer, August 1999, 38-45

[HMB] Hayashi, I., Maeda, T., Bastian, A., Jain, L.C.: Generation of fuzzy decision trees by fuzzy ID3 with adjusting mechanism of AND/OR operators, Proc. 1988 Int. Conf on Fuzzy Systems (FUZZ-IEEE 98) (1998) 681-685

[HKR] Hoppner, F., Klawonn, F., Kruse, R., Runkler, T.: Fuzzy Cluster Analysis, J. Wiley, Chichester, 1999

[ISN] Ichihashi, H., Shirai, T., Nagasaka, K., Miyoshi, T.: Neuro-fuzzy ID3: a method of inducing fuzzy decision trees with linear programming for maximizing entropy and an algebraic method for incremental learning, Fuzzy Sets and Systems **81/1** (1996) 157-167

[ImM] Imielinski, T., Manilla, H.: A Database Perspective on Knowledge Discovery, Comm. ACM **38/11** (1996) 58-64

[Ja1] Janikow, C. Z.: Fuzzy decision trees: issues and methods, IEEE Trans. on Systems, Man and Cybernetics, part B (Cybernetics) **28/1** (1998) 1-14

[Ja2] Janikow, C.Z.: Fuzzy partitioning with FID 3.1, In: Proc. 18th Int. Conf. of the North American Fuzzy Information Processing Society, NAFIPS -99, New York, NY, June 10-12 (1999) 467-471

[PVK] Kacprzyk, J., S. Zadrożny: Data Mining via Fuzzy Querying over Internet, In: Pons, O., Amparo Vila M., Kacprzyk, J. (Eds.), Knowledge Management in Fuzzy Databases, Physica-Verlag, Heidelberg, 1999, 211-233

[KR] Kaufman, L., Rousseeuw, P.J.: Finding Groups in Data, J. Wiley, New York, 1990

[KPS] Komorowski, J., Pawlak, Z., Polkowski, L., and Skowron, A.: Rough sets: A tutorial, in: S.K. Pal and A. Skowron (eds.), Rough fuzzy hybridization: A new trend in decision--making, Springer-Verlag, Singapore (1999) 3-98

[KK] Krishnapuram, R., Keller, J.: A possibilistic approach to clustering, IEEE Trans. on Fuzzy Systems **1** (1993) 98-110

[Lan] Langley, P., Simon, H.A., Bradshaw, G.L., Żytkow, J.M.: Scientific Discovery, Computational Explorations of the Creative Processes, The MIT Press, Cambridge, Massachusetts, 1987

[LI3] Lin, T. Y., Cercone, N. (eds.): Rough Sets and Data Mining - Analysis of Imperfect Data. Kluwer Academic Publishers, Boston, London, Dordrecht, 1997

[Mi] Mitchell, T.: Machine Learning, McGraw Hill, New York, 1997

[Mo1] Mollestadt, T.: A Rough Set Approach to Data Mining: Extracting a Logic of Default Rules from Data, Ph.D. Thesis, Norwegian University of Science and Technology, Trondheim, 1997

[Ng1] Nguyen, H.S.: Efficient SQL-learning Method for Data Mining in Large Data Bases, Proc. Sixteenth International Joint Conference on Artificial Intelligence (IJCAI'99) (1999) 806-811

[NSS] Nguyen, H.S., Skowron, A., Synak, P.: Discovery of Data Patterns with Applications to Decomposition and Classification Problems. In: [PSc], 55-97

[NS2] Nguyen, H.S.: Discretization of real value attributes: Boolean reasoning approach. Ph.D. Dissertation, Warsaw University (1997) 1-90

[NH1] Nguyen, S.H., Skowron, A., Synak, P.: Discovery of data patterns with applications to decomposition and classification problems. In: [PSc], 55-97

[Or1] Orłowska, E. (ed.): Incomplete Information: Rough Set Analysis. Physica-Verlag, Heidelberg, 1997

[Pa1] Pawlak, Z.: Rough Sets - Theoretical Aspects of Reasoning about Data, Kluwer Academic Publishers, Dordrecht, 1991

[PaS3] Pawlak, Z., Skowron, A.: Rough Set Rudiments, Bulletin of the International Rough Set Society 3/4 (1999) 181-185

[Pal] Pal, S.K., Skowron, A. (eds.): Rough--Fuzzy Hybridization: A New Trend in Decision Making, Springer-Verlag, Singapore, 1999

[PE1] Pedrycz, W.: Computational intelligence: An introduction. CRC Press, Boca Ratou, 1998

[Pe2] Pedrycz, W.: Conditional Fuzzy C - Means, Pattern Recognition Letters **17** (1996) 625 - 632

[Pe3] Pedrycz, W.: Conditional fuzzy clustering in the design of radial basis function neural networks, IEEE Trans. on Neural Networks **9/4** (1998) 601-612

[Pe4] Pedrycz, W.: Fuzzy set technology in knowledge discovery, Fuzzy Sets and Systems **3** (1998) 279-290

[PS0] Polkowski, L., Skowron, A.: Rough mereology, Proceedings of the 8th International Symposium on Methodologies for Intelligent Systems (ISMIS'94), Charlotte, October 1994, Lecture Notes in Artificial Intelligence 869, Springer-Verlag, Berlin, 1994, 85-94

[LP] Polkowski, L., Skowron, A.: Rough mereology in information systems. A case study: Qualitative spatial reasoning, in: [PTL] (in print)

[PS1] Polkowski, L., Skowron, A.: Rough mereology: A new paradigm for approximate reasoning. In: International Journal of Approximate Reasoning **15/4** (1996) 333-365

[PSa] Polkowski, L., Skowron, A.: Towards Adaptive Calculus of Granules, In: [ZaK] **1**, 201-227

[PSb] Polkowski, L., Skowron, A. (eds.): Rough Sets in Knowledge Discovery vol **1-2**, Physica-Verlag, Heidelberg, 1998

[PS2] Polkowski, L., Skowron, A.: Rough mereological foundations for design, analysis, synthesis, and control in distributive systems. Information Sciences International Journal **104/1-2** Elsevier Science, New York (1998) 129-156

[PTL] Polkowski, L., Tsumoto, S., Lin, T.Y. (eds.): Rough Sets: New Developments, Physica-Verlag, Heidelberg (2000) (in print)

[RI1] Rissanen, J.J.: Modeling by Shortest Data Description, Automatica **14** (1978) 465-471

[RS1] Roddick, J.F., Spiliopoulou, M.: A Bibliography of Temporal, Spatial, and Temporal Data Mining Research, Newsletter of the Special Interest Group (SIG) on Knowledge Discovery & Data Mining **1/1**, http://research.microsoft.com/datamine/sigkdd/, 34-38

[Sch] Schultze-Kremer, S.: Discovery in human genome project, Communications of the ACM **42/11** (1999), 62-64

[SKA] Selman, B., Kautz, H., McAllester, D.: Ten Challenges in Propositional Reasoning and Search, Proc. IJCAI'97, Japan

[ShL] Shrager, J., Langley, P.: Computational Approaches to Scientific Discovery, In: Shrager, J., Langley, P. (Eds.), Computational Models of Scientific Discovery and Theory Formation, Morgan Kaufmann, San Mateo, 1990, 1-25

[SN1] Skowron, A., Nguyen, H.S.: Boolean Reasoning with Some Applications in Data Mining., 3rd European Conference of Principles and Practice of Knowledge Discovery in Databases, September 15-18, 1999, Prague, Czech Republic, Lecture Notes in Artificial Intelligence **1704**, Springer-Verlag, Heidelberg (1999) 107-115

[SR1] Skowron, A., Rauszer, C.: The Discernibility Matrices and Functions in Information Systems. In: [Sl1], 311-362

[SST] Skowron, A., Stepaniuk, J., Tsumoto, S.: Information Granules for Spatial Reasoning, Bulletin of the International Rough Set Society **3/4**, 1999, 147-154

[Sl1] Słowiński, R. (ed.): Intelligent Decision Support. Handbook of Applications and Advances of the Rough Set Theory, Kluwer Academic Publishers, Boston, London, Dordrecht 1992

[ST1] Stepaniuk, J.: Knowledge Discovery by Rough Set Methods, in: [PTL] (in print)

[SzM] Szczuka, M.: Symbolic and neural network methods for classifiers construction, Ph.D. Thesis, Warsaw University, 2000

[Zyt] Troxel, M., Swarm, K., Zembowicz, R., Zytkow, J.M.: Concept hierarchies: A restricted form of knowledge derived from regularities, Proceedings of the 8th International Symposium on Methodologies for Intelligent Systems (ISMIS'94), Charlotte, October 1994, Lecture Notes in Artificial Intelligence 869, Springer-Verlag, Berlin, 1994, 448-457

[Ts1] Tsumoto, S., et al. (eds.): The Fourth Internal Workshop on Rough Sets, Fuzzy Sets and Machine Discovery, proceedings. November 6-8, The University of Tokyo, 1996

[UOH] Umano, M., Okamoto, H., Hatono, I., Tamura, H., Kawachi, F., Umedzu, S., Kinoshita, J.: Generation of fuzzy decision trees by fuzzy ID3 algorithm and its application to diagnosis by gas in oil, Proc. 1994 Japan-USA Symposium on Flexible Automation (1994) 1445-1448

[V-P] Valdéz-Pérez, R.E.: Discovery Tools for Science Apps, Comm ACM **42/11** (1999), 37-41

[Ya1] Yao, Y.Y.: Combination of rough and fuzzy sets based on alpha-level sets. In: [Li3], 301-321

[Za1] Zadeh. L.A.: Fuzzy sets. Information and Control **8** (1965) 333-353

[Za2] Zadeh. L.A.: Fuzzy logic = computing with words. IEEE Trans. on Fuzzy Systems **4** (1996) 103-111

[Za4] Zadeh, L. A. (1979) Fuzzy sets and information granularity, In: M.M. Gupta, R.K. Ragade, R.R. Yager, eds., *Advances in Fuzzy Set Theory and Applications*, North Holland, Amsterdam, 3-18

[ZaK] Zadeh, L.A., Kacprzyk, J. (eds.): Computing with Words in Information/Intelligent Systems vol.1-2, Physica-Verlag, Heidelberg, 1999.

[ZI1] Ziarko, W.: Variable precision rough sets model. Journal of Computer and Systems Sciences **46/1** (1993) 39-59