1 Towards Discovery of Relevant Patterns from Parameterized Schemes of Information Granule Construction

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Abstract. The paper introduces a step in developing a foundation for approximate reasoning from experimental data to conclusions in natural language. We consider schemes of approximate reasoning consisting of information granules from different information sources. By tuning the parameters of schemes, the information granules representing patterns relevant to given tasks can be generated. Approximation granules, rough-fuzzy granules and granule decomposition based on background knowledge are briefly considered in this paper. The contribution of this paper is the presentation of an approach to obtaining relevant patterns from parameterized schemes of information granule construction.

Keywords: rough set theory, information granulation, pattern, granular computing, schemes of approximate reasoning, decomposition, information fusion

1.1 Introduction

Information granulation belongs to intensively studied topics in soft computing (see, e.g. [29], [30], [31]). One of the recently emerging approaches to deal with information granulation is based on information granule calculi (see, e.g. [15], [22], [18]). The development of such calculi is important for making progress in many areas like object identification by autonomous systems (see e.g. [3], [28]), web mining (see e.g. [6]), approximate reasoning based on information granules (see, e.g., [23], [11], [21], [18], [19]) or spatial reasoning (see e.g. [26], [4], [2], [12]). In particular, reasoning methods using background knowledge as well as knowledge extracted from experimental data (e.g. sensor measurements) represented by concept approximations [3] are important for making progress in such areas.

Schemes of approximate reasoning (AR-schemes, for short) are obtained by means of relevant patterns for a given task decomposition of the identified or classified complex objects. The problem of AR-schemes deriving is closely related to perception [1], [31], [27], [17], [13].

In the paper we assume AR-schemes define parameterized operations on information granules. We discuss problems of tuning these parameters to derive from them relevant granules included in (or close to) target concepts to a satisfactory degree. Target concepts are assumed to be incomplete and/or vague.

One can distinguish two kinds of considered parts (represented, e.g. by subformulas or sub-terms) of AR-schemes. The first type of scheme parts is represented by expressions from a language, called the *domestic* language L_d, that has known semantics (consider, for example, a given information system [10]). The second type of scheme parts is from a language, called *foreign* language L_f (e.g. natural language, that has semantics definable only in an approximate way (e.g., by means of patterns extracted using rough, fuzzy, rough–fuzzy or other approaches). For example, the parts of the second kind of scheme can be interpreted as soft properties of sensor measurements [3].

For a given expression e, representing a given scheme that consists of subexpressions from L_f we propose to search for relevant approximations in L_d of the foreign parts from L_f and next to derive patterns from the whole expression after replacing the foreign parts by their approximations. This can be a multilevel process, i.e. we are facing problems of discovered pattern propagation through several domestic-foreign layers [25].

Let us consider three strategies of patterns construction from schemes. The first strategy entails searching for relevant approximations of parts using a rough set approach. This means that each part from L_f can be replaced by its lower or upper approximation with respect to a set B of attributes. The approximation is constructed on the basis of relevant data table [3]. With the second strategy parts from L_f are partitioned into a number of sub-parts corresponding to cuts (or set theoretical differences between cuts) of fuzzy sets representing vague concepts and each sub-part is approximated by means of rough set methods. The third strategy is based on searching from given languages for patterns sufficiently included in foreign parts. In all cases, the extracted approximations replace foreign parts in the scheme and candidates for global patterns are derived as output from the AR-scheme obtained after the replacement. Searching for relevant global patterns is a complex task because many parameters should be tuned, e.g. the set of relevant features used in approximation, relevant approximation operators, the number and distribution of objects from the universe of objects among different cuts and so on. We plan to use evolutionary techniques for relevant pattern searching to obtain optimal parameters with respect to the quality of synthesized patterns.

We propose an approach for extracting from data patterns relevant to the target concept α. In the first step the expression $Pattern(\beta_1, \ldots, \beta_k)$ representing a scheme including foreign parts $\beta_1, \ldots, \beta_k$ for some integer $k > 0$ is discovered. It represents the decomposition of the global object (situation).

Two kinds of measures are considered:

1. *inclusion measure* $\nu_p(\gamma, \beta)$ expressing that γ is included in β in degree at least p;
2. closeness measure $cl_p(\gamma, \beta)$ expressing that γ and β are close in degree at least p.

For more information on inclusion and closeness measures the reader is referred to [15], [22]).

The main goal is to extract from data rules of approximate reasoning like:

- **if** $\nu_{q_1}(\gamma_1, \beta_1)$ **and** ... **and** $\nu_{q_k}(\gamma_k, \beta_k)$
  then $\nu_q(Pattern(\beta_1/\gamma_1, \ldots, \beta_k/\gamma_k), \alpha)$;
- **if** $cl_{q_1}(\gamma_1, \beta_1)$ **and** ... **and** $cl_{q_k}(\gamma_k, \beta_k)$
  then $cl_q(Pattern(\beta_1/\gamma_1, \ldots, \beta_k/\gamma_k), \alpha)$;

where parameters $q_1, \ldots, q_k \in [0,1]$ chosen for a given $q \in [0,1]$ can be extracted from data by decomposition of q. The decomposition process is multilayered and strongly depends on the scheme structure and available experimental data [15], [25].

1.2 Approximation Granules

We use standard notation (see [10]). In particular by $IS = (U, A)$ we denote an information system, (a, v) denotes a descriptor defined by the attribute a and its value v, α denotes Boolean combination of descriptors, and $[\alpha]_{IS}$ (or $[\alpha]_A$) its meaning in IS, i.e. the set of all objects from U satisfying α. The lower and upper approximations of $X \subseteq U$ are denoted by $\underline{A}X$ and $\overline{A}X$, respectively.

An elementary granule (in IS) is any pair $(\alpha, [\alpha]_{IS})$ (for simplicity of considerations we assume $[\alpha]_{IS} \neq \emptyset$).

One can extend the set of elementary granules assuming that if α is any Boolean combination of descriptors over A, then $(\overline{B}\alpha, \overline{B}[\alpha]_{IS})$ and $(\underline{B}\alpha, \underline{B}[\alpha]_{IS})$ are elementary granules too, for any $B \subseteq A$.

The inclusion and closeness are the basic concepts related to information granules [15], [22]. Using them one can measure the closeness of the constructed granule to the target granule and robustness of the construction scheme with respect to deviations of information granules being components of the construction. For details and examples of closeness relations the reader is referred to [15], [22]. Here, we present only the necessary introductory remarks. Let us consider an example. For $X, X' \subseteq U$ let us assume, e.g. $\nu(X, X') = card(X \cap X')/card(X)$ if $X \neq \emptyset$ and 1, otherwise. By $\nu_p(X, X')$ we denote $\nu(X, X') \geq p$. We assume $cl_p(X, X')$ if and only if $\nu_p(X, X')$ and $\nu_p(X', X)$. Closeness relation can be extended to information granules; $cl_p(g, g')$ means that the information granules g, g' are close in degree at least p [22].

It is worth mentioning that one can consider instead of Boolean propositional connectives other operations to create granule descriptions. Operations on information granules have been discussed, e.g. in [15], [16], [22]. Among these operation are set theoretical operation of union and intersection, operations defined by data tables, operations generating classifiers, etc. They are used to define complex information granules from elementary ones. Other operations can be used to define granules corresponding to classifiers. Let us observe that inclusion (closeness) measures can be used to define new granules being approximations or generalizations of existing ones. Assume g, h are given information granules, and ν_p are inclusion measures for $p \in [0,1]$. A (h,p)- *approximation* of g is an information granule $\nu_{h,p}$ represented by a set $\{h' : \nu_1(h', h) \wedge \nu_p(h', g)\}$. Now one can easily define the lower and upper approximations of information granules [20].

Let us now denote by L_e^B the language of elementary granules consisting of Boolean combinations of expressions of the following two kinds:

- descriptors over attributes from $B \subseteq A$;

- formulas of the form $\overline{B}\alpha$, $\underline{B}\alpha$ where α is a Boolean combination of descriptors over attributes from A.

The formulas from L_e^B describe the syntax of B-elementary granules in IS. By Sem_B we denote the semantics of formulas from L_e^B, i.e. the function assigning to any formula $\alpha \in L_e^B$ the set $[\alpha]_{IS}$ of objects satisfying α in IS. Any pair $(\alpha, [\alpha]_{IS})$, where $\alpha \in L_e^B$ is called a B-elementary granule in IS.

Any elementary granule $(\alpha, [\alpha]_{IS})$ in IS defines the granule

$$g = (Approx_B(\alpha), \{[\beta]_{IS} : \beta \in Approx_B(\alpha)\})$$

where $Approx_B(\alpha) \subseteq L_e^B$ is the set of all formulas which can be obtained from α, in a finite number of steps, by application of the following rule:

if $\beta \notin L_e^B$ is a subformula of α (not prefixed by $\overline{B}$ or $\underline{B}$) then it is replaced by $\overline{B}\beta$ or $\underline{B}\beta$.

Hence, g consists of all granules obtained from α by replacing foreign parts by their approximations using attributes from B. An important task is to define among components of g those which are relevant for matching the target information granule, i.e. those patterns returned by the scheme which are sufficiently close to (or included in) the target granule.

Now, let us consider a target D-elementary granule $g_t = (\delta, [\delta])_{IS}$ where $D \subseteq A$ and $D \cap B = \emptyset$. For a given degree p one can consider a new granule being in a sense a part of

$$g = (Approx_B(\alpha), \{[\beta]_{IS} : \beta \in Approx_B(\alpha)\})$$

and defined by

$$g' = (Approx'_B(\alpha), \{[\beta]_{IS} : \beta \in Approx'_B(\alpha)\})$$

where $Approx'_B(\alpha)$ is the set of all formulas γ from $Approx_B(\alpha)$ close to g_t (or included in g_t) in degree at least p, i.e. $cl_p([\gamma]_{IS}, [\delta]_{IS})$ (or $\nu_p([\gamma]_{IS}, [\delta]_{IS})$).

The target information granule g_t is approximated by means of granules sufficiently close to it. The granules are extracted from a given data sample represented by information system IS. Approximating granules are constructed not only from descriptors over B but also by means of the B-lower and B-upper approximations of some formulas consisting of attributes not necessarily from B.

Certainly, one should develop methods for inducing target concept approximation considered on extensions of IS. One possible way to induce target concept approximation is to use evolutionary methods [7] to solve the problem by optimization of parameters involved in the approximate description of the target concept.

Some parameters to be optimized express the approximation quality of concepts represented by sub-formulas with approximation operators. These coefficients can be also used to measure the robustness of the target concept approximation.

Let us consider some examples. Any pair

$$((\alpha, [\alpha]_{IS}), (\beta, [\beta]_{IS}))$$

of elementary granules in IS such that $[\alpha]_{IS} \subseteq [\beta]_{IS}$, $[\alpha]_{IS} = \underline{A}X$ and $[\beta]_{IS} = \overline{A}X$, for some $X \subseteq U$, can be considered as a granule. Let us call it approximation granule. If α is any Boolean combination of descriptors over A (not necessarily over B) then

$$g = ((\underline{B}\alpha, \underline{B}[\alpha]_{IS}), (\overline{B}\alpha, \overline{B}[\alpha]_{IS}))$$

is called the B-approximation granule. By $Sem(g)$ we denote a pair $(\underline{B}[\alpha]_{IS}, \overline{B}[\alpha]_{IS})$.

For simplicity of notation we use only the semantic part $Sem(g)$ of g to denote the granule g, if this not lead to misunderstanding.

Let $g = ((\alpha, [\alpha]_{IS}), (\beta, [\beta]_{IS}))$ and $g' = ((\alpha', [\alpha']_{IS}), (\beta', [\beta']_{IS}))$ be two approximation granules. They are close in degree at least p (in IS), in symbols $cl_p(g, g')$ if and only if

- $cl_p([\alpha]_{IS}, [\alpha']_{IS})$;
- $cl_p([\beta]_{IS} - [\alpha]_{IS}, [\beta']_{IS} - [\alpha']_{IS};)$
- $cl_p(U - [\alpha]_{IS}, U - [\alpha']_{IS})$.

This means that two approximation granules g, g' are close in degree at least p (in IS) if and only if the lower approximations, upper approximations and boundary regions defined by them are close in degree at least p, respectively. The degree of closeness describes the quality of approximation of sets X satisfying the above conditions. Another coefficient which can be used for expressing closeness of approximation granules to sets they approximate is

$$Q(g) = \frac{card([\alpha]_{IS})}{card([\beta]_{IS})}$$

where $((\alpha, [\alpha]_{IS}), (\beta, [\beta]_{IS}))$ is an approximation granule in IS.

The quality of a sequence $g = (g_1, ..., g_k)$ of approximation granules $g_i = ((\alpha_i, [\alpha_i]_{IS}), (\beta_i, [\beta_i]_{IS}))$ where $i = 1, \ldots, k$ is defined by

$$Q(g) = \frac{card(\bigcup_{i=1}^{k} [\alpha_i]_{IS})}{card(\bigcup_{i=1}^{k} [\beta_i]_{IS})}.$$

1.3 Rough–Fuzzy Granules

In this section we will discuss briefly approximation schemes of granules and methods for extracting from them relevant patterns in case when they include fuzzy concepts as foreign parts. We propose to use rough set approach to define in a constructive way approximations of fuzzy concepts. The rough set approximations of the fuzzy cuts are used in searching for constructive definition of fuzzy sets. We use the cut approximations to derive from schemes patterns relevant for the target concept. In the process of searching for high quality patterns evolutionary techniques can be used.

Let $DT = (U, A, d)$ be a decision table with the decision being the restriction to the objects from U of the fuzzy membership function $\mu : U \rightarrow [0, 1]$. Consider reals $0 < c_1 < \ldots < c_k$ where $c_i \in (0, 1]$ for $i = 1, \ldots, k$. Any c_i defines c_i-cut by $X_i = \{x \in U : \mu(x) \geq c_i\}$. Assume $X_0 = U, X_{k+1} = X_{k+2} = \emptyset$.

A *rough–fuzzy granule* (*rf–granule*, for short) corresponding to $(DT, c_1, \ldots, c_k)$ is any granule $g = (g_0, \ldots, g_k)$ such that for some $B \subseteq A$

1. $Sem_B(g_i) = (\underline{B}(X_i - X_{i+1}), \overline{B}(X_i - X_{i+1}))$
   for $i = 0, \ldots, k$;
2. $\overline{B}(X_i - X_{i+1}) \subseteq (X_{i-1} - X_{i+2})$
   for $i = 1, \ldots, k$.

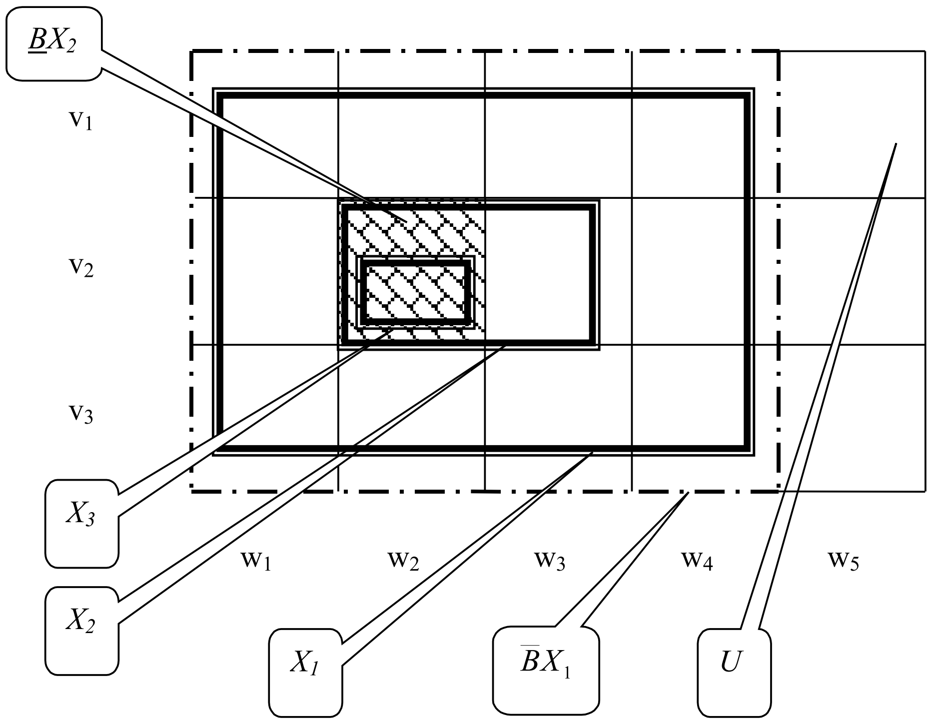

Fig. 1.1. Example of Rough–Fuzzy Granule

In Figure 1.1 an example of rough–fuzzy granule is presented. We assume that there are three cuts, i.e. $k = 3$ and we obtain the following sequence of sets $\emptyset = X_5 = X_4 \subset X_3 \subset X_2 \subset X_1 \subset X_0 = U$. We also assume that a set of attributes B is equal to $\{a_1, a_2\}$. There are three possible values v_1, v_2, v_3 of attribute a_1 and five possible values w_1, w_2, w_3, w_4, w_5 of attribute a_2. Thus we obtain fifteen indiscernibility classes described by descriptor conjunctions of the form $a_1 = v_i \wedge a_2 = w_j$, where $i = 1, 2, 3$ and $j = 1, 2, 3, 4, 5$. The lower approximation of X_3 is equal to the empty set and the upper approximation of X_3 is equal to the lower approximation of X_2. The upper approximation of X_2 is equal to the lower approximation of X_1.

Any function $\mu^* : U \to [0,1]$ satisfying the following conditions:

1. $\mu^*(x) = 0$ for $x \in U - \overline{B}X_1$;
2. $\mu^*(x) = 1$ for $x \in \underline{B}X_k$;
3. $\mu^*(x) = c_{i-1}$ for $x \in \underline{B}(X_{i-1} - X_i)$ and $i = 2, \ldots, k-1$;
4. $c_{i-1} < \mu^*(x) < c_i$ for $x \in (\overline{B}X_i - \underline{B}X_i)$, $i = 1, \ldots, k$, and $c_0 = 0$;

is called a B-approximation of μ.

Now one can choose of the lower or upper approximations of parts, i.e. the set theoretical differences between successive cuts, and propagate them along the scheme in searching for relevant patterns. Another strategy is to propagate the global approximation of foreign fuzzy concepts through the scheme.

The approach presented here can be generalized. One can consider approximations of relevant patterns instead of cut approximations and use them for deriving rules of approximate reasoning like those discussed in Introduction. One of such methods for extracting patterns from data is presented in [9]. The method is based on decomposition of data tables with respect to attributes aiming at finding the largest Cartesian products of local patterns sufficiently included in the target concept. In the following section we discuss a decomposition problem based on background knowledge. This problem is of a great importance in classification of situations by autonomous systems on the basis of sensor measurements [28].

1.4 Granule Decomposition

In this section, we discuss briefly a granule decomposition problem. This is one of the basic problems in synthesis of approximate schemes of reasoning from experimental data. We restrict our considerations to the case of information granule decomposition supported by background knowledge.

Assume knowledge base consists of a fact expressing that if two objects belong to concepts C_1 and C_2, then the object constructed out of them by means of a given operation f belongs to the concept C (see Figure 1.2)provided that the two objects satisfy some constraints. However, we can only approximate these concepts on the basis of available data. Using a (generalized) rough set approach [20] one can assume that it is given an inclusion measure v_p for $p \in [0,1]$ making it possible to estimate the degree of inclusion of data patterns Pat, Pat_1, and Pat_2 from languages L, L_1, and L_2 in the concepts C, C_1, and C_2, respectively. Patterns included in a satisfactory degree p in a concept are classified as belonging to its lower approximation while those included to a degree less than a preset threshold $q \leq p$ are classified as belonging to its complement. The decomposition problem is a searching problem for patterns Pat of *high* quality (e.g. supported by a large number of objects) and included in a satisfactory degree in the target concept C. These patterns are obtained by performing a given operation f on some input patterns Pat_1 and Pat_2 (from languages L_1 and L_2, respectively) sufficiently included in C_1 and C_2, respectively.

One can develop a searching method for such patterns Pat based on tuning of inclusion degrees p_1, p_2 of input patterns Pat_1, Pat_2 in C_1, C_2, respectively, to obtain patterns Pat (constructed from Pat_1, Pat_1 by means of a given operation f) included in C in a satisfactory degree p and of acceptable quality (e.g. supported by the number of objects larger than a given threshold).

Assume degrees p_1, p_2 are given. There are two basic steps of searching procedures for relevant pairs of patters (Pat_1, Pat_2) :

1. Searching in languages L_1 and L_2 for sets of patterns included in degree at least p_1 and p_2 in concepts C_1 and C_2, respectively.
2. Selecting from sets of patterns generated in Step 1 satisfactory pairs of patterns.

We would like to add some general remarks on the above steps.

One can see that our method is based on a decomposition of degree p into degrees p_1 and p_2 under some constraints. In Step 2 we search for a relevant constraint relation R between patterns. The goal is to extract the following approximate rule of reasoning:

if

$R(Sem(Pat_1), Sem(Pat_2)) \wedge$
$\nu_{p_1}(Sem(Pat_1), C_1) \wedge$
$\nu_{p_2}(Sem(Pat_2), C_2)$

then

$\nu_p(f(Sem(Pat_1) \times Sem(Pat_2)), C) \wedge$
$Quality_t(f(Sem(Pat_1) \times Sem(Pat_2)))$

where p is a given inclusion degree, t - a threshold of pattern quality measure $Quality_t$, f- operation on objects (patterns), Pat- target pattern, C, C_1, C_2-given concepts, R, p_1, p_2 are expected to be extracted from data and (Pat_1, Pat_2) is satisfying R (in our case R is represented by a finite set of pattern pairs).

One can consider soft constraint relations R_r where $r \in [0,1]$ is a degree of truth to which the constraint relation holds.

Two sets P_1, P_2 are returned as the result of the first step. They consist of pairs (*pattern, degree*) where *pattern* is included in C_1, C_2, respectively in degree at least *degree*.

These two sets are used to learn the relevant relation R. We outline two methods.

The first one is based on an experimental decision table (U, A, d) where U is a set of pairs of discovered patterns in the first step; $A = \{deg_1, deg_2\}$ consists of two attributes such that $deg_i((Pat_1, Pat_2))$ is equal to the degree to which Pat_i is at least included in C_i for $i = 1, 2$; the decision d has value p to which the granule composed by means of operation f from (Pat_1, Pat_2) is at least included in C. From this decision table the decision rules of a special form are induced:

$$\textbf{if } deg_1 \geq p_1 \wedge deg_2 \geq p_2 \textbf{ then } d \geq p$$

where (p_1, p_2) is a minimal degree pair such that if $p_1' \geq p_1$ and $p_2' \geq p_2$ then the decision rule obtained from the above rule by replacing p_1', p_2' instead of p_1, p_2, respectively, is also true in the considered decision table.

A version of such a method has been proposed in [14]. The relation R consists of the set of all pairs (Pat_1, Pat_2) of patterns with components included in C_1, C_2, respectively in degrees $p_1' \geq p_1$, $p_2' \geq p_2$ where p_1, p_2 appear on the left hand side of some of the generated decision rules.

The second method is based on another experimental decision table (U, A, d) where objects are triplets $(x, y, f(x, y))$ composed out of objects x, y and the result

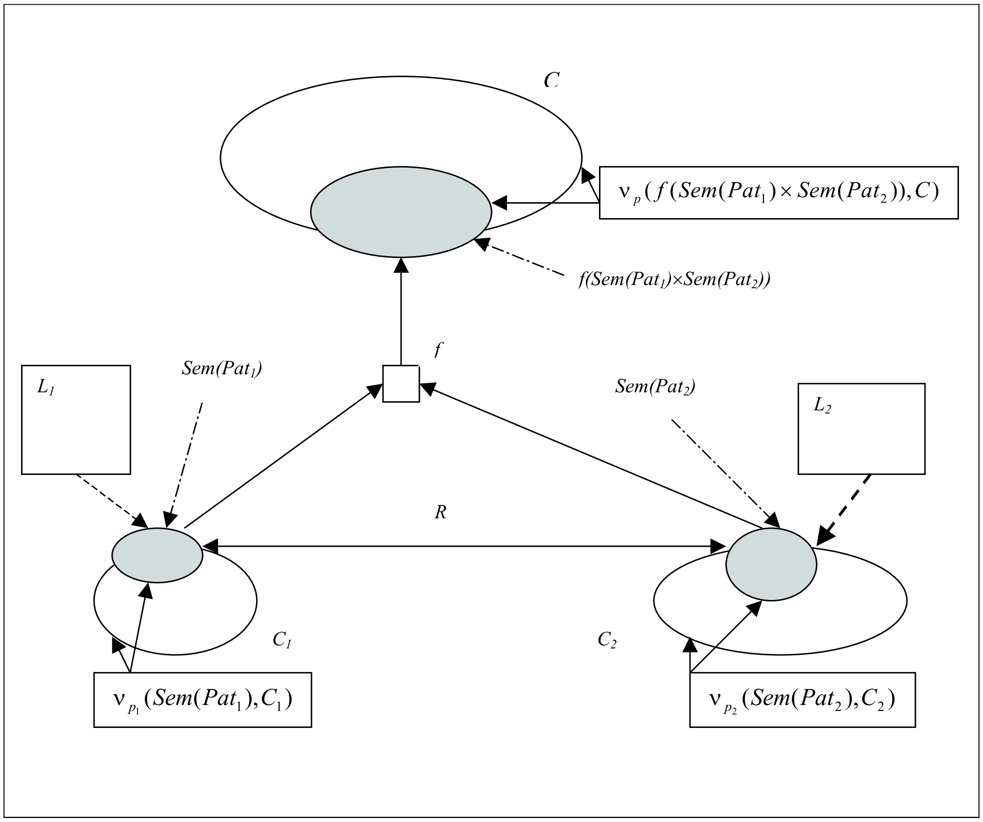

Fig. 1.2. Granule Decomposition Based on Background Knowledge

of f on arguments x, y; attributes from A describe features of arguments of objects and the decision d is equal to the degree to which the elementary granule corresponding to the description of $f(x, y)$ by means of attributes is at least included in C. This table is extended by adding new features being characteristic functions a_{Pat_i} of patterns Pat_i discovered in the first step. Next the attributes from A are deleted and from the resulting decision table the decision rules of a special form are induced:

$$\textbf{if } a_{Pat_1} = 1 \wedge a_{Pat_2} = 2 \textbf{ then } d \geq p$$

where if Pat_1, Pat_2 are included in C_1, C_2, in degree at least p_1, p_2, respectively and Pat'_1, Pat'_2 are included in C_1, C_2 in degree $p'_1 \geq p_1$ and $p'_2 \geq p_2$, respectively then a decision rule obtained from the above rule by replacing Pat'_1, Pat'_2 instead of Pat_1, Pat_2 is also true in the considered decision system.

The decision rules again describe constraints specifying the constraint relation R.

Certainly, in searching procedures one should also consider constraints for the pattern quality.

The searching methods discussed in this section return local granule decomposition schemes. These local schemes can be composed using techniques discussed in [15]. The received schemes of granule construction (which can be also treated as approximate reasoning schemes) have also the following property: if the input granules are sufficiently close to input concepts then the output granule is sufficiently included in the target concept provided this property is preserved locally [15].

Searching for relevant patterns for information granule decomposition can be based on methods for tuning parameters of rough set approximations of fuzzy cuts or concepts defined by differences between cuts (see Section 1.3). In this case pattern languages consist of parameterized expressions describing the rough set approximations of *parts* of fuzzy concepts being fuzzy cuts or differences between cuts. Hence, an interesting research direction related to the development of new hybrid rough-fuzzy methods arises aiming at developing algorithmic methods for rough set approximations of such parts of fuzzy sets relevant for information granule decomposition.

Conclusions

We have discussed an approach for extracting relevant patterns from parameterized schemes of information granule construction consisting of parts from different information sources. The schemes can be also treated as schemes of approximate reasoning built on the basis of perception by means of information granule calculi. Relevant output patterns (information granules) can be obtained by tuning of the scheme parameters. We have emphasized the necessity of approximation (in an accessible language) of information granules being parts of schemes and expressed in another language called foreign language. In our further study we plan to develop evolutionary searching techniques for optimal parameters of information granule construction schemes extracted from data and background knowledge.

In our further study, we plan to implement the proposed strategies and test them on the above-mentioned real-life data. This will require the following steps: (i) development of ontologies for considered applications, (ii) further development of methods for extracting productions from data on the basis of decomposition, and (iii) synthesis methods for AR-schemes from productions. These methods will make it possible to reason by means of sensor measurements along inference schemes over ontologies (i.e., inference schemes over some standards) by means of attaching to them AR-schemes from background knowledge (including ontologies) and experimental data.

Acknowledgements

The research of Andrzej Skowron has been supported by the State Committee for Scientific Research of the Republic of Poland (KBN) research grant 8 T11C 025 19 and partially by the Wallenberg Foundation grant. The research of Jarosław

Stepaniuk has been supported by the State Committee for Scientific Research of the Republic of Poland (KBN) research grant 8 T11C 025 19. The research of James Peters has been supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) research grant 185986.

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