On Some Conflict Models and Conflict Resoultions

Andrzej Skowron[1], *Rafał Deja*[2]

[1]Institute of Mathematics, Warsaw University Banacha 2, 02-097 Warsaw, Poland,
Email: skowron@mimuw.edu.pl
[2]Tokarskiego 4/14, 40-749 Katowice, Poland, Email: rd@alta.pl

1 Introduction

The importance of multi-agents systems, models of agents' interaction is increasing nowadays as distributed systems of computers started to play a significant role in society. An interaction occurs when two or more agents, which have to act in order to attain their objectives, are brought into a dynamic relationship. This relationship is the consequence of the limited resources which are available to them in a situation. If the number of resources is insufficient to attain agents' goals it often comes into the conflicts. This can happen in almost all industrial activities requiring distributed approach, such as network control, the design and manufacture of industrial products or the distributed regulation of autonomous robots. However, distributed systems is only one from many different areas where a conflict can arise and where it is worth to apply computer aided conflict analysis. Just to mention some human activities like business, government, political or military operations, labour-management negotiations etc. etc.

In the paper, we explain the nature of conflict and we define the conflict situation model in a way to encapsulate the conflict components in a clear manner. We propose some methods to solve the most fundamental problems related to conflicts. The model introduced in this paper is an enhancement of the model proposed by Pawlak in papers e.g. [23, 24].

2 Pawlak Model

The simple model introduced by Pawlak [21] forms the basis for the model presented in this paper. Pawlak defines a conflict situation by an information system; $S = (U, A)$, where U - is a nonempty, finite set called the universe; elements of U are called objects (here agents), A - is a nonempty, finite set of attributes (issues).

Every attribute $a \in A$ is a map, $a : U \to V_a$, where the set V_a is the value set of a; elements of V_a are referred to as opinions, i.e. $a(x)$ or a_x is the opinion of an agent x about an issue a. The domain of each attribute (for the conflict analysis model) is restricted to three values only, i.e. $V_a = -1, 0, 1$, which means that an agent is against, neutral and favourable toward the issue, respectively.

Furthermore the binary relations of coalition, neutrality and conflict are defined in a set of agents [24]. For example two agents ag and ag' are in conflict $R_a^-(ag, ag')$ on attribute a when their views on issue a are opposite – formally iff function $\phi_a(ag, ag') = -1$.

$$\phi_a(ag, ag') = \begin{cases} 1 & \text{when } a(ag)a(ag') = 1 \quad \text{or } ag = ag' \\ 0 & \text{when } a(ag)a(ag') = 0 \quad \text{and } ag \neq ag' \\ -1 & \text{when } a(ag)a(ag') = -1 \end{cases}$$

Example 1. Let us consider a conflict between an employer and employees represented by two different trade unions TU1 and TU2. The example of conflict is taken from the author's observation, though it has been simplified to present the defined notions rather then resolve a real conflict. Employers are interested mainly in factory profit, good investment level and, maybe, worker's satisfaction. Job attributes considered for the workers from TU1 are compensation and work conditions. The most important factors for the workers from TU2 are salary, social care policy but also the level of employment (some reductions has been proposed). We can think about these attributes quite generally, for example, compensation can consist of the worker's salary and all his income but it also can include the repeated profit division like the social fund. Similarly worker's conditions include a modern and safe work place and in addition a nice team and development possibilities. We analyse the conflict presented in this example more deeply in the whole paper.

Let us choose the issues for the Pawlak model (agents are voting on):

a – increasing the employees' incomes,

b – improving the work conditions,

c – increasing the social care, warranty of the current level of employment,

d – increasing the factory profit by reducing the costs of work (reductions in employment),

e – increasing the level of investment to grow up the factory profit.

Then, the information table (Table 1), where ag_3 is the employer, ag_1 and ag_2 represents TU1 and TU2 respectively, can describe the conflict situation.

	a	b	c	d	e
ag_1	-1	0	-1	1	1
ag_2	1	1	0	0	-1
ag_3	0	1	1	-1	-1

Table 1. The conflict situation in Pawlak Model

Analysis of conflicts described by the Pawlak model is restricted to outermost conclusions like finding the most conflicting attributes or the coalitions of agents if more than two take part in the conflict [9]. Because in the Pawlak model the reason for the conflict cannot be determined, there is no way to specify the situation for avoiding the conflict. Moreover, we cannot be sure that the issues the agents vote represent the issues each agent takes care of. In the real world, views on the issues to vote are consequences of the decision taken, based on the local issues, the current state and some background knowledge. Therefore, the Pawlak model is enhanced here by adding to this model some local aspects of conflicts.

3 New Model

The information about the local states U_{ag} of an agent ag can be represented in the form of an information table, creating the agent ag's information system $I_{ag} = (U_{ag}, A_{ag})$, where $a : U_{ag} \to V_a$ for any $a \in A_{ag}$ and V_a is the value set of attribute a. We assume: $V_{ag} = \bigcup\limits_{a \in A_{ag}} V_a$

Any local state $s \in U_{ag}$ is explicitly described by its information vector $Inf_{A_{ag}}(s)$, where $Inf_{A_{ag}}(s) = \{(a, a(s)) : a \in A_{ag}\}$. The set $Inf_{A_{ag}}(s) : s \in U_{ag}\}$ is denoted by $INF_{A_{ag}}$ and it is called the information vector set of ag. We assume that sets $\{A_{ag}\}$ are pairwise disjoint, i.e., $A_{ag} \cap A_{ag'} = \emptyset$ for $ag \neq ag'$. This condition emphasizes that any agent is describing the situation in its own way. The manner of understanding the same world by each agent can be completely different. Relationships among attributes of different agents will be defined by constraints as shown in section 3.3.

Example 2 illustrates local states for the labour-management conflict.

3.1 Subjective evaluation of local states (similarity of states)

Every agent evaluates the local states. The subjective evaluation corresponds to an order (or partial order) of the states in the agent information table. We assume that the function e_{ag} called the target function, assigns an evaluation score to each state; let for example $e_{ag} : U_{ag} \to R[0, 1]$. The states with score 1 are mostly preferred by the agent as target states, while the states with score 0 are not acceptable. Maximal elements (determined by a partial order) can be interpreted as those, which are targets of the agent, i.e., the agent wants to reach them e.g. in a negotiation process.

More precisely the agent ag's set of goals (targets) denoted by T_{ag} is defined as the set of target states of ag, which means $T_{ag} = \{s \in U_{ag} : e_{ag}(s) > \mu_{ag}\}$, and μ_{ag} is the acceptance level, chosen by the agent ag – it is subjective which evaluation level is acceptable by the agent.

The state evaluation can also help us to find the state similarity (see e.g. [26] for references on similarity in rough set investigations). For any $\varepsilon > 0$ and $s \in U_{ag}$, we define *ε-neighbourhood* of s by: $\tau_{ag,\varepsilon}(s) = \{s\prime \in U_{ag} : |e_{ag}(s) - e_{ag}(s')| \leq \varepsilon\}$

The family $\{\tau_{ag,\varepsilon}(s)\}_{s \in U_{ag}}$ defines a tolerance relation $\tau_{ag,\varepsilon}$ in $U_{ag} \times U_{ag}$ by $s\tau_{ag,\varepsilon}s\prime$ iff $s\prime \in \tau_{ag,\varepsilon}(s)$.

Example 2. Let us consider the situation described in Example 1. *Ag* consists of three agents: ag_1 – TU1, ag_2 –TU2 and ag_3 – the employer. Table 2 shows agent's ag_1 states, i.e., views on local issues (attributes) a, b and the state subjective evaluation. Consequently Table 3 shows agent's ag_2 states. Where attributes denotes: a – compensation, b – work conditions, s – salary, t – social care, u – the level of employment and for the employer: k – company profit, l – level of investment, m – workers satisfaction.

local states	a	b	e_{ag_1}
s_1	2	2	1
s_2	2	1	$\frac{2}{3}$
s_3	1	2	$\frac{2}{3}$
s_4	1	1	0
s_5	1	0	0

Table 2. Agent ag_1 local states with subjective evaluation.

local states	s	t	u	e_{ag_2}
s_1	2	1	2	1
s_2	2	2	1	1
s_3	2	1	1	$\frac{2}{3}$
s_4	1	2	1	$\frac{1}{3}$
s_5	1	1	2	$\frac{1}{3}$
s_6	1	1	1	0
s_7	2	0	1	0
s_8	0	1	2	0

Table 3. Agent ag_2 local states with subjective evaluation.

For simplicity, let us assume that attributes' domains for all agents are the same, and values belong to the set $V = \{0, 1, 2\}$. One can interpret the values from set V as *small*, *medium* and *high* levels, respectively. For example, the state s_1 of the agent ag_1 expresses a high level of compensation and high level of work conditions.

In the considered situation, the minimal acceptable level of evaluation by the agents will be, e.g., a score greater than $\frac{1}{3}$. Accordingly sets of goals of agents ag_1 and ag_2 are as follows: $T_{ag_1} = \{s_1, s_2, s_3\}$ and $T_{ag_2} = \{s_1, s_2, s_3\}$

The set of goals can also be presented in the propositional form. The information table with scores is going to be converted to the decision table in which the decision 1 means that the state belongs to the set of goals, while 0 that it does not. Then the rules for decision 1 are found (for the method of rule generation see e.g. [18, 34]). The decision table of an agent ag_3 with the threshold $\frac{1}{3}$

is constructed and presented in Table 4.

local states	k	l	m	e_{ag_3}	decision d
s_1	2	2	2	1	1
s_2	1	2	2	2/3	1
s_3	1	1	2	1/3	0
s_4	1	1	1	1/3	0
s_5	2	0	1	0	0

Table 4. Decision table of agent ag_3 local states.

Rule for $d = 1 : l_2 \vee (k_2 \wedge m_2) \rightarrow \ d_1$
Rule for $d = 0 : l_0 \vee l_1 \vee m_1 \rightarrow d_0$

The decision class for d=1 describes the agent ag_3 local set of goals: $t_{ag_3} = l_2 \vee (k_2 \vee m_2)$. The decision rules generated here are based on minimal relative reducts to classify invisible objects (states) well. This rough set method is especially useful in conflict analysis, when we cannot expect precise and complete information. Similarly the local set of goals of agents ag_1 and ag_2 can be described respectively by: $t_{ag_1} = a_2 \vee b_2$ and $t_{ag_2} = (s_2 \wedge t_1) \vee (s_2 \wedge t_2) \vee (s_2 \wedge u_2)$

Distance function A tolerance relation τ describes similarity of states according to the subjective evaluation. However, it is necessary to describe the state similarity according to differences between values of attributes.

Similarity of states from U_{ag} can be often defined as follows. We assume that for any $a \in A_{ag}$ there is a distance function: $d_a : U_{ag} \times U_{ag} \rightarrow \ R+$

Next we define the distance function $d : U_{ag} \times U_{ag} \rightarrow \ R+$ by $d(s, s\prime) = F(d_{a_1}(s, s\prime), ..., d_{a_m}(s, s\prime))$ where $A_{ag} = \{a_1, ..., a_m\}$ and $F : R^m_+ \rightarrow \ R+$ is a function like e.g. $F(r_1, ..., r_m) = \sqrt{r_1^2 + ... + r_m^2}$

The function F depends on the problem and should be chosen reflecting the problem specificity.

3.2 Situations

Let us consider a set Ag consisting of n agents ag_1,...,ag_n. A *situation* of Ag is any element of the Cartesian product $S(Ag) = \prod_{i=1}^{n} INF^*(ag_i)$, where $INF^*(ag_i)$ is the set of all possible information vectors of the agent ag_i, defined by $INF^*(ag) = \{f : A_{ag} \rightarrow \bigcup_{a \in A_{ag}} V_a(ag) : f(a) \in V_a(ag)\}$ The situation $\bar{s}(Ag) \in S(Ag)$ corresponding to the global state $\bar{s} = (s_1, ..., s_n) \in U_{ag_1} \times ... \times U_{ag_n}$ is defined by $(Inf_{A_{ag_1}}(s_1), ..., Inf_{A_{ag_n}}(s_n))$.

3.3 Constraints

Constraints are described by some dependencies among local states of agents. Without any dependencies, any agent could take the state freely. If there is no influence of a given agent on states of other agents – there is no conflict at all. Dependencies among local states of agents come from the bound on the number of resources (any kind of a resource may be considered, e.g. water on Golan Hills see [23] or an international position [19], everything that is essential for agents). Constraining relations are introduced to express which local states of agents can coexist in the (global) situation. More precisely, constraints are used to define a subset S(Ag) of global situations.

Constraints restrict the set of possible situations to admissible situations satisfying constraints. We will consider only admissible situations (shortly, situations) in the rest of the paper.

Example 4. The following dependencies restrict the set of situations and are constraints in our example. Attribute names here stand for the variables corresponding to attribute values. Constants here have been taken experimentally to express relationships and to allow comparison of any two variables.

1. $a > 0$ (compensation must be middle at least)
2. $u > 0$ (the level of employment must be at least middle too)
3. $l + m \geq u$ (the level of employment depends on the investment level and workers satisfaction)
4. $2 + a \geq s + t$ (compensation includes the salary and the social care)
5. $2 + m = a + b$ (workers' satisfaction comes from a good compensation and work conditions)
6. $3 \cdot k \geq a + l + s + t$ (division of profit – a very simple case, i.e., the company uses its current profit for all expenses)

Constraints above can be converted to propositional formulas ($f_{\varphi 1}, f_{\varphi 2} \cdots f_{\varphi 6}$) accordingly. For example the equation $a > 0$ yields the formula $\mathrm{f}_{\varphi 1}$=$a_1 \vee a_2$. The conjunction of formulas $f_{\varphi} = f_{\varphi 1} \wedge f_{\varphi 2} \wedge f_{\varphi 3} \wedge f_{\varphi 4} \wedge f_{\varphi 5} \wedge f_{\varphi 6}$ defines all admissible situations in our example. As already mentioned, constraints describe the situations that are admissible i.e. all local states can coexist in the admissible situation. For example, the situation $a = 2, b = 2, s = 2, t = 2, u = 2, k = 2, l = 2, m = 2$ is not admissible because of constraint 6.

3.4 Situations evaluation

Like local states the global situations are evaluated too. The score assigned to each situation is taken into account when looking for consensus. It can reflect the agents preferences (subjective states evaluation) and consensus consist on looking for the situation preferred by the majority of agents.

The agents tend to attain the best states without taking care about the global good. However, the negotiators experience shows that the real, stable consensus can only be found when the global good is considered. Thus the other, important way of scoring the situation is the expert judgement. One can think of the United Nation Organisation as an example of an expert in the military conflicts.

Objective evaluation of situations We assume there is a function $q : S(Ag) \rightarrow R[0,1]$, called the quality function, which assigns a score to each situation. The set of situations satisfying a given level of quality t is defined by: $Score_{Ag}(t) = S \in S(Ag) : q(S) \geq t$

Example 5. Table 5 presents some situations scored by an expert in our conflict.

situations		b	s	t	u	k	l	m	q(S)	decision
S_1	1	2	1	2	1	2	2	2	1	1
S_2	1	2	2	2	2	2	2	1	1	1
S_3	1	1	2	1	1	1	1	1	$\frac{2}{3}$	1
S_4	1	2	1	2	1	1	2	2	$\frac{2}{3}$	1
S_5	1	2	1	2	1	2	1	2	$\frac{1}{3}$	0
S_6	1	2	2	2	2	1	2	1	0	0
S_7	1	2	1	1	1	2	2	0	$\frac{1}{3}$	0
	...	...	...	...	...	...	...	...		
S_{19} (current)	1	2	2	2	1	2	0	1	0	0
S_{20}	1	2	1	2	1	1	2	1	0	0
S_{21}	1	2	1	1	2	2	1	1	$\frac{1}{3}$	0
S_{22}	2	0	1	1	1	2	2	1	0	0

Table 5. Objective situations evaluation (decision table).

The assumed quality level is (decision1) $t = \frac{2}{3}$. Thus the set $Score_{Ag}(\frac{2}{3})$ is described by the following formula (we have generated the decision class based on minimal relative reducts).

$a_1k_2l_2m_2 \vee a_1s_2k_2l_2m_1 \vee b_2s_2k_2l_2m_1 \vee a_1u_2k_2l_2m_1 \vee b_2u_2k_2l_2m_1 \vee a_1t_1k_2l_2m_1 \vee b_2t_1k_2l_2m_1 \vee k_1l_1m_1 \vee k_1l_2m_2 \vee t_1u_2k_2l_2m_1 \vee s_1u_2k_2l_2m_1 \vee \Leftrightarrow d_1$

Agents preferences The second way of situation scoring is to transfer the subjective evaluation of local states into the situations. The global preference function of situation S corresponding to the global state $\bar{s} = (s_1, ..., s_n)$ can be defined by:

$p(\bar{s}) = (s_1, s_2, ..., s_n)) = F(e_{ag_1}(s_1), e_{ag_2}(s_2), ..., e_{ag_n}(s_n))$ where F is a suitable function e.g.: $F(r_1, ..., r_m) = \sum_{i=1}^{m} r_i$ [17].

Consequently the set of all preferred situations can be defined by: $S_{pref_{Ag}}(h) = \{S : p(S) \geq h\}$,where h is a chosen *preference level.*

The function proposed here is very simple, however the global evaluation can be described in any suitable form (also non-linear) like in the form of decision rules.

4 System with constraints

The multi-agent system, with local states for each agent defined and the global situations satisfying constraints, will be called the system with constraints. We denote our system with constraints by M_{Ag}.

5 Conflict definition

In previous section the system with constraints has been specified. In such systems, conflict can be defined on several different levels. Below we define some of them.

5.1 Local conflict

The agent *ag* is in the ϵ-local conflict in a state s iff s does not belong to the ϵ-neighbourhood of $s\prime$, for any $s\prime$ from the set of *ag-targets* where ϵ is a given threshold. Local conflict for an agent *ag* arises from the low level of subjective evaluation of the current state by *ag*. It can be expressed differently that state s does not belong to the ε-environs of the set of goals T_{ag} i.e.: $s \notin \bigcup_{s' \in T_{ag}} \tau_{ag,\epsilon}(s')$, where $\tau_{ag,\epsilon}(s') = \{s'' : s'' \tau_{ag,\epsilon} s'\}$.

5.2 Global conflict (based on an expert evaluation)

A situation S is called *t-objectively conflicting* for Ag where t is a given threshold iff S does not belong to the set $Score_{Ag}(t)$. When the current situation is conflicting for Ag then agents from Ag are in the objective global conflict. The difference between the situation score and the given threshold can be treated as a global conflict degree, i.e.,

$$Cg^t_{Ag}(S) = \begin{cases} t - q(S) \text{ when } t > q(S) \\ \quad 0 \text{ otherwise} \end{cases},$$

where t is the given threshold and q is the quality function.

Example 8. In the discussed example of labour-management conflict, let us take $t = \frac{2}{3}$. There is a global conflict in the current situation S_{19} with a tension $Cg^{\frac{2}{3}}_{Ag}(S_{19}) = 2/3 - 0 = 2/3$.

5.3 Global conflict (based on agents preferences)

Consequently, a situation S is called *t-conflicting* for Ag where t is a given threshold iff S does not belong to the set $Spref_{Ag}(t)$. When the current situation is conflicting for Ag then agents from Ag are in the global conflict. The difference between the situation score and the given threshold can be treated as this kind of conflict degree, i.e.,

$$Cp^t_{Ag}(S) = \begin{cases} t - p(S) \text{ when } t > p(S) \\ \quad 0, \text{ otherwise} \end{cases},$$

where t is the given threshold and p is the global preference function.

6 Analysis

The introduced above conflict model gives us possibility, first to understand and, then, to analyse different kinds of conflicts. Particularly, the most fundamental problem can be widely investigated, that is, the possibility to achieve the consensus. As in everyday life, the consensus can be found on several levels and under some conditions. Can be based on the situations objective and/or subjective evaluation, can also include local agents preferences. Only the consensus problem on local preferences is studied in this paper other can be found in [8]. We propose Boolean reasoning [5] and Rough Set methodology [22] for all analysis. The main idea of Boolean reasoning is to encode the optimisation problem, by corresponding Boolean function f_π in such a way that any prime implicant of f_π states a solution of π. The elementary Boolean formula is usually obtained here by transforming the information table into the decision table, generating rules (minimal with respect of number of attributes on left side) and determining the description of decision class [34]. From the elementary formulas the final formula describing the problem is shaped.

Unfortunately calculating prime implicants of such formulas is usually a hard-computational problem [18]. Therefore depending on the formula, some simple strategies or eventually quite complex heuristics must be used to resolve the problem in real time.

6.1 Consensus problem on local and global level

In this section a conflict analysis is proposed where local information tables and the set of local goals are taken into consideration.

INPUT

The system with constraints M_{Ag} defined in Section 3.

t - an acceptable threshold of the objective global conflict for Ag.

OUTPUT

All situations with the objective evaluation reduced to degree at most t, and without local conflict for any agent. (it is required that any new situation is constructed in a way that all local states in this situation are favourable for the agents).

ALGORITHM

The algorithm is based on verification of global situations from $Score_{Ag}(t)$ with the local set of goals of agents and constraints. The problem is described by the formula f: $f = \bigwedge_{ag \in Ag} t_{ag} \wedge f_C \wedge f_\varphi$, where t_{ag} describes the set of goals of the agent ag, and f_C describes $Score_{Ag}(t)$ and f_φ the constraints. The formula $f_C \wedge f_\varphi$ representing all admissible situations without the global conflict regarding the threshold t.

Example 7. The way of constructing the formulas t_{ag}, f_φ, f_C has been presented in Example 3, Example 4, Example 5 respectively. Including the formulas

describing the local set of goals the formula f has the following form:
$f = (a_2 \wedge b_2) \vee (s_2t_1 \wedge s_2t_2 \wedge s_2u_2) \vee (l_2 \wedge k_2m_2) \vee f_C \vee f_\varphi$

After reduction we have four prime implicants and four solutions – non-conflicting situations:
$f = a_1b_2s_2t_1u_1k_2l_2m_1 \wedge a_1b_2s_2t_1u_2k_2l_2m_1 \wedge a_1b_2s_2t_1u_1k_2l_2m_2 \wedge a_1b_2s_2t_1u_2k_2l_2m_2$

7 Calculation strategies

The reduction (calculating prime implicants) of formulas described in the previous section can be exhausted or time consuming. In the consensus problems we have to verify the local goals $f_1, ..., f_n$ against the formula of core situations (belonging to $Score_{Ag}(t)$) and constraints f_φ. This usually yields long formulas looked like this: $f = f_1 \vee f_2 \vee ... f_n \vee f_C \vee f_\varphi$.

Simple strategies can be based on the Boolean algebra rules. First, the absorption rule has to be considered when choosing the formulas to calculate the formulas conjunction - a shorter formula can strongly reduce the longer formula being an extension of the shorter one. Thus the order of formulas conjunction is important and appropriate strategy can be built [8].

Another important notice, which can be useful in calculation strategy is that the result (if exists) is a disjunction of f_φ components. On the other hand formulas $f_1 \ldots f_n$ consist of components based on different Boolean variables (set of attributes $\{A_{ag}\}$ are pairewise disjoint). Thus any prime implicant must contain a component from each agent formula $f_1 \ldots f_n$. In the following strategy we are verifying all remaining formulas (component after component) against agents formulas. The considered component can be removed if it does not contain any component of formula describing a given agent set of targets. More precisely the strategy of *preliminary reduction* can be described by the following algorithm.

Let $f[][]$ denotes the two dimensional array for storing formulas – for each formula the components are stored. Let first nAg formulas of array $f[][]$ describe agents sets of targets.

```
for $i = nAg$ to formulas no do
    for $j = 1$ to components no of $f[i]$ do
        bRemove =false;
        for $z = 1$ to $nAg$ do
            for $k = 1$ to components no of $f[z]$ do // for one agent
                if $f[i][j] \wedge f[z][k] = f[z][k]$ then
                    break; // do not remove this component
                if $f[i][j] \wedge f[z][k] = \emptyset$ then
                    bRemove = true;
            endfor;
            if $k \leq$ components no of $f[z]$ then
                break;
        endfor;
        if bRemove then
            remove component $f[i][j]$;
```

```
        endif;
    endfor;
endfor;
// conjunction of remaining formulas
for $i = 1$ to formulas no - 1 do
    $f$[formulas no]=$f[i]f$[formulas no];
endfor;
printf[formulas no]; // result-reduced formula
```

The preliminary reduction strategy allows in a time depended on components number (pessimistic calculation time $\mathrm{O}(n^2)$) to check and possibly remove these components, which are normally reduced during conjunction. However in the algorithm without preliminary reduction the number of components can at the start exponentially grow during conjunction. Unfortunately not all components which have to be reduced are removed within the proposed algorithm.

Example 8 The formula describing problem consensus on local and global level is as follows:

$f = (a_2 \wedge b_2) \vee (s_2t_1 \wedge s_2t_2 \wedge s_2u_2) \vee (l_2 \wedge k_2m_2) \vee (a_1k_2l_2m_2 \wedge a_1s_2k_2l_2m_1 \wedge b_2s_2k_2l_2m_1 \wedge a_1u_2k_2l_2m_1 \wedge b_2u_2k_2l_2m_1 \wedge a_1t_1k_2l_2m_1 \wedge b_2t_1k_2l_2m_1 \wedge k_1l_1m_1 \wedge k_1l_2m_2 \wedge t_1u_2k_2l_2m_1 \wedge s_1u_2k_2l_2m_1) \vee f_\varphi$

How the algorithm is functioning will be shown on the core formula as an example. Because the first component $a_1k_2l_2m_2$ consists of component l_2 of agent ag_3 it cannont be removed. Similarly with next components. The only component which can be reduced is $k_1l_1m_1$ because $k_1l_1m_1 \vee (l_2 \wedge k_2m_2) = \emptyset$.

8 Conclusions

We have presented and discussed the extension of the Pawlak conflict model. The understanding of the underlying local states as well as constraints in the given situation is the basis for any analysis of our world. The local goals and the evaluation of the global situation are observed as factors defining the strength of the conflict and can suggest the way to reach the consensus.

The fundamental consensus problem has been analysed in the paper. Then, Boolean reasoning and rough set theory has been successfully applied for solving presented problem.

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