Information Granules:

Towards Foundations for Spatial and Temporal Reasoning

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Abstract

The aim of the paper is to present an approach for spatial and temporal reasoning in the granular computing framework. We introduce some basic notions related to granular computing, namely the information granule syntax and semantics as well as the inclusion and closeness (similarity) relations of granules. Different information sources (units, agents) are equipped with two kinds of operations on information granules. They are: operations possessed by agents transforming tuples of information granules definable by a given agent into information granules definable by the agent and unary approximation operations for computing by agents information granule approximations delivered by other agents. More complex granules are constructed by means of these operations and approximation operations from some input information granules. The construction of information granules is described by expressions called terms. We discuss one of the fundamental problems of spatial and temporal reasoning namely a problem of synthesis of robust terms, i.e., descriptions of information granules, satisfying a given specification.

Introduction

In the recent years a lot of research has been done in the Qualitative Spatial and Temporal Reasoning area (see e.g. [11], [3], [4], [15], [16]). Several international workshops and conferences have been organized, the European network SPACENET [16] focusing on Qualitative Spatial Reasoning was created. Qualitative Spatial and Temporal Reasoning methods are important for solving of many real-life problems like visual object recognition including the interpretation and integration of visual information, cognitive maps and path finding, spatial planning, geographical information systems or determination of the $3-D$ structure of molecules. These methods should take into account that the spatial and temporal reasoning should be carried out under uncertainty from incomplete and complex knowledge. Several approaches have been developed to deal with some aspects of spatial and temporal reasoning. However, a widely accepted foundations for such reasoning have not been discovered yet [2], [3].

Methods for qualitative spatial and temporal reasoning are closely related to a paradigm Computing with Words recently formulated by Lotfi Zadeh [18], [19]. Several attempts have been made to develop foundations for computing with words. Among them there is rapidly growing area of granular computing aiming to develop models for computing with information granules.

They are two basic notions for granular computing: information granule and calculus on information granules [8].

Notions of information granule [18], [19], [8] and information granule similarity (inclusion or closeness) are very useful for knowledge discovery. Informally speaking, information granules can be treated as linked collections of objects drawn together by the criteria of indiscernibility, similarity or functionality [18], [19].

The exact interpretation between granule languages of different information sources (agents) often does not exist. Hence (rough) inclusion and closeness of granules are considered instead of their equality. For example, the left and right hand sides of association rules [1] describe granules. The *support* and *confidence* coefficients specify the inclusion degree of granule represented by the formula on the left hand side into the granule represented by the formula on the right hand side of the association rule.

We generalize a simple notion of granules represented by attribute value vectors as well as closeness (inclusion) relation to the case of complex information granules representing concepts. In many application areas like Spatial and Temporal Reasoning or Knowledge Discovery and Data Mining there is a need for algorithmic methods to discover much more complex information granules and relations between them than investigated so far. We discuss examples of more complex information granules. The basic relations between information granules are (rough) inclusion and closeness. The relations between information granules can be defined by extension of the relations defined on parts of the information granules. One can expect that these linked granules have similar properties. We will look for a formalization of this claim. In the consequence we obtain that similar constructions of information granules lead to similar complex information granules, which are satisfying the same input specification.

Reasoning in distributed environment requires a construction of interfaces between agents (information sources or units) to enable effective learning by agents of concepts definable by other agents. We emphasize the fact concepts definable by some agents can be not definable by the other ones. However, they can be approximately definable. In the paper we suggest one solution based on exchanging views of agents on objects with respect to a given concept. An agent delivering concept is giving positive and negative examples (objects) with respect to a given concept. The agent receiving this information can describe objects using its own attributes. In this way a data table (called a *decision table*) is created and the receiving agent can extract the approximate concept description. Our solution is based on rough set approach. We propose to use the parameterized approximation spaces [13], [14] to allow appropriate tuning of concept perception by agents.

An analogous method can be used in case of the *customer-agent* (agent specifying tasks) searching for a top-level cooperating agent (*root-agent*). The customer-agent is presenting examples and counterexamples of objects with respect to her/his concept. Agents approximate the concept specified by customer-agent and an agent returning the best approximation of the customer-agent concept is chosen to be the root agent. The goal of cooperating agents is to produce a concept sufficiently close (or included) to the concept specified by the customer-agent. This concept has to be constructed from some elementary concepts available for agents called *input-agents* (inventory-agents or leaf-agents). This is realized by searching for an agent scheme [7], [9]. The schemes are represented in the paper by expressions called *terms*.

Our approach can be treated as an approach for synthesis of information granules describing some complex concepts. In the process of synthesis rough set methods are used, especially the approximation operations.

An important property of information granules for spatial and temporal reasoning is that we deal with complex information granules formed as mentioned above by families of simpler information granules linked together by the criteria of indiscernibility, similarity or functionality [18], [19].

There are at least two reasons to develop such approach. The first is related to compression of necessary information to be stored. The second follows from a need that concepts represented by terms should be robust with respect to parameter deviations. We consider here a notion of satisfiability in a degree developed in rough mereological approach [7], [8] to deal with the problem. This together with parameterized approximation spaces enable us to define robust constructions, represented by terms. Informally speaking, the robustness of term closeness (or inclusion) representing information granules means that the closeness (inclusion) properties is preserved in a satisfactory degree under small deviations of input granules or operations used in their construction. In particular this allows formulating a scheme for deducing the global robustness of terms from local robustness of their sub-terms.

This paper contains an extension of results presented in [13], [14]. We adopt some basic ideas developed in rough mereological approach (see e.g. [7], [8], [9], [10]).

Rough Sets and Approximation Spaces

We recall general definition of approximation space [12], [17].

Definition

A *parameterized approximation space* is a system $AS_{\#,\$} = (U, I_{\#}, \nu_{\$})$, where

- U is a non-empty set of objects,
- $I_{\#} : U \to P(U)$ is an uncertainty function and $P(U)$ denotes the powerset of U,
- $\nu_{\$} : P(U) \times P(U) \to [0,1]$ is a rough inclusion function.

The uncertainty function defines for every object x a set of similarly described objects. A constructive definition of uncertainty function can be based on the assumption that some metrics (distances) are given on attribute values. For example, if for some attribute $a \in A$ a metric $\delta_a : V_a \times V_a \to [0, \infty)$ is given, where V_a is the set of all values of attribute $a \in A$ then one can define the following uncertainty function:

$y \in I_a^{f_a}(x)$ if and only if $\delta_a(a(x), a(y)) \le f_a(a(x), a(y))$,

where $f_a : V_a \times V_a \to [0, \infty)$ is a given threshold function.

A set $X \subseteq U$ is *definable in* $AS_{\#,\$}$ if it is a union of some values of the uncertainty function.

The rough inclusion function defines the degree of inclusion between two subsets of U [12], [8].

The lower and the upper approximations of subsets of U are defined as follows.

Definition

For a parameterized approximation space $AS_{\#,\$} = (U, I_{\#}, \nu_{\$})$ and any subset $X \subseteq U$ the lower and the upper approximations are defined by

$LOW(AS_{\#,\$}, X) = \{x \in U : \nu_{\$}(I_{\#}(x), X) = 1\}$,

$UPP(AS_{\#,\$}, X) = \{x \in U : \nu_{\$}(I_{\#}(x), X) > 0\}$, respectively.

Approximations of concepts (sets) are constructed on the basis of background knowledge. Obviously, concepts are also related to unseen so far objects. Hence it is very useful to define parameterized approximations with parameters tuned in the searching process for approximations of concepts. This idea is crucial for construction of concept approximations using rough set methods. In our notation $\#,\$$ are denoting parameter vectors, which can be tuned in the process of concept approximation.

Syntax and Semantics of Information Granules

The presented above definition of approximation space can be treated as a semantic part of the approximation space definition. Usually there is also specified a set of formulas Φ expressing properties of objects. Hence we assume that together with the approximation space $AS_{\#,\$}$ there are given

- a set of formulas Φ over some language,
- semantics $\|\bullet\|$ of formulas from Φ, i.e., a function from Φ into the power set $P(U)$.

Let us consider an example [6]. We define a language L_{IS} used for elementary granule description, where $IS = (U, A)$ is an information system. The syntax of L_{IS} is defined recursively by

1. $(a \in V) \in L_{IS}$, for any $a \in A$ and $V \subseteq V_a$.
2. If $\alpha \in L_{IS}$ then $\neg\alpha \in L_{IS}$.
3. If $\alpha, \beta \in L_{IS}$ then $\alpha \wedge \beta \in L_{IS}$.
4. If $\alpha, \beta \in L_{IS}$ then $\alpha \vee \beta \in L_{IS}$.

The semantics of formulas from L_{IS} with respect to an information system $IS = (U, A)$ is defined recursively by

1. $\|a \in V\|_{IS} = \{x \in U : a(x) \in V\}$.
2. $\|\neg\alpha\|_{IS} = U - \|\alpha\|_{IS}$.
3. $\|\alpha \wedge \beta\|_{IS} = \|\alpha\|_{IS} \cap \|\beta\|_{IS}$.
4. $\|\alpha \vee \beta\|_{IS} = \|\alpha\|_{IS} \cup \|\beta\|_{IS}$.

A typical method used by the classical rough set approach [6] for constructive definition of the uncertainty function is the following: for any object $x \in U$ there is given information $Inf_A(x)$

(information vector, attribute value vector of x) which can be interpreted as conjunction $EF_B(x)$ of selectors $a = a(x)$ for $a \in A$ and the set $I_\#(x)$ is equal to $\|EF_B(x)\|_{IS} = \|\wedge_{a\in A}\, a = a(x)\|_{IS}$. One can consider a more general case taking as possible values of $I_\#(x)$ any set $\|\alpha\|_{IS}$ containing x. Next from the family of such sets the resulting neighborhood $I_\#(x)$ can be selected or constructed. One can also use another approach by considering more general approximation spaces in which $I_\#(x)$ is a family of subsets of U [5].

In the sequel we will consider several general kinds of information granules. We present now their syntax and semantics. In the following section we discuss the inclusion and closeness relations for granules.

Elementary granules. In an information system $IS = (U, A)$, elementary granules are defined by $EF_B(x)$, where $EF_B(x)$ is a conjunction of selectors of the form $a = a(x)$, $B \subseteq A$ and $x \in U$. For example, the meaning of an elementary granule $a = 1 \wedge b = 1$ is defined by

$$\|a = 1 \wedge b = 1\|_{IS} = \{x \in U : a(x) = 1 \,\&\, b(x) = 1\}.$$

Sequences of granules. Let us assume that S is a sequence of granules and the semantics $\|\bullet\|_{IS}$ in IS of its elements have been defined. We extend $\|\bullet\|_{IS}$ on S by $\|S\|_{IS} = \{\|g\|_{IS}\}_{g\in S}$.

Example

Granules defined by rules in information systems are examples of sequences of granules. Let IS be an information system and let (α, β) be a new information granule received from the rule

if α **then** β

where α, β are elementary granules of IS. The semantics $\|(\alpha, \beta)\|_{IS}$ of (α, β) is the pair of sets $(\|\alpha\|_{IS}, \|\beta\|_{IS})$.

Sets of granules. Let us assume that a set G of granules and the semantics $\|\bullet\|_{IS}$ in IS for granules from G have been defined. We extend $\|\bullet\|_{IS}$ on the family of sets $H \subseteq G$ by $\|H\|_{IS} = \{\|g\|_{IS} : g \in H\}$.

Example

One can consider granules defined by sets of rules. Assume that there is a set of rules $Rule\_Set = \{(\alpha_i, \beta_i) : i = 1, \ldots, k\}$. The semantics of $Rule\_Set$ is defined by

$$\|Rule\_Set\|_{IS} = \{\|(\alpha_i, \beta_i)\|_{IS} : i = 1, \ldots, k\}.$$

Example

One can also consider as set of granules a family of all granules $(\alpha, Rule\_Set(DT_\alpha))$, where α belongs to a given subset of elementary granules.

Example

Granules defined by sets of decision rules corresponding to a given situation α are also examples of sequences of granules. Let $DT = (U, A \cup \{d\})$ be a decision table and let α be an elementary granule of $IS = (U, A)$ such that $\|\alpha\|_{IS} \neq \emptyset$. Let $Rule\_Set(DT_\alpha)$ be the set of decision rules (e.g. in minimal form) of the decision table $DT_\alpha = (\|\alpha\|_{IS}, A \cup \{d\})$ being the restriction of DT to objects satisfying α. We obtain a new granule $(\alpha, Rule\_Set(DT_\alpha))$ with the semantics

$$\|(\alpha, Rule\_Set(DT_\alpha))\|_{DT} = (\|\alpha\|_{IS}, \|Rule\_Set(DT_\alpha)\|_{DT}).$$

This granule describes a decision algorithm applied in the situation characterized by α.

Extension of granules defined by tolerance relation. We present examples of granules obtained by application of a tolerance relation.

Example

One can consider extension of elementary granules defined by tolerance relation (reflexive and symetric relation). Let $IS = (U, A)$ be an information system and let τ be a tolerance relation on elementary granules of IS. Any pair (α, τ) is called a τ - *elementary granule*. The semantics $\|(\alpha, \tau)\|_{IS}$ of (α, τ) is the family $\{\|\beta\|_{IS} : (\beta, \alpha) \in \tau\}$.

Let us consider granules defined by rules of tolerance information systems. Let $IS = (U, A)$ be an information system and let τ be a tolerance relation on elementary granules of IS. If

if α **then** β

is a rule in IS then the semantics of a new information granule $(\tau : \alpha, \beta)$ is defined by $\|(\tau : \alpha, \beta)\|_{IS} = \|(\alpha, \tau)\|_{IS} \times \|(\beta, \tau)\|_{IS}$.

Example

We consider granules defined by sets of decision rules corresponding to a given situation α in tolerance decision tables. Let $DT = (U, A \cup \{d\})$ be a decision table and let τ be a tolerance on elementary granules of $IS = (U, A)$. Now, any granule $(\alpha, Rule\_Set(DT_\alpha))$ can be considered as a representative of information granule cluster

$$(\tau : (\alpha, Rule\_Set(DT_\alpha)))$$

with the semantics

$$\|(\tau : (\alpha, Rule\_Set(DT_\alpha)))\|_{DT} = \{\|(\beta, Rule\_Set(DT_\beta))\|_{DT} : (\beta,\alpha) \in \tau\}.$$

Granule Inclusion and Closeness

In this section we will discuss inclusion and closeness of different information granules introduced in the previous section. Let us mention that the choice of inclusion or closeness definition depends very much on area of application and data analyzed. This is the reason that we have decided to introduce a separate section with this more subjective part of granule semantics.

The inclusion relation between granules G, G' of degree at least p will be denoted by $\nu_p(G, G')$. Similarly, the closeness relation between granules G, G' of degree at least p will be denoted by $cl_p(G, G')$. By p we denote a vector of parameters (e.g. positive real numbers).

A general scheme for construction of hierarchical granules and their closeness can be described by the following recursive meta-rule:

if granules of order $\leq k$ and their closeness have been defined

then the closeness $cl_p(G, G')$ (at least in degree p) between granules G, G' of order $k+1$ can be defined by applying an appropriate operator F to closeness values of components of G, G'.

Elementary granules. We have introduced the simplest case of granules in information system $IS = (U, A)$. They are defined by $EF_B(x)$, where EF_B is a conjunction of selectors of the form $a = a(x)$, $B \subseteq A$ and $x \in U$. Let $G_{IS} = \{EF_B(x) : B \subseteq A \;\&\; x \in U\}$. In the standard rough set model [6] elementary granules describe indiscernibility classes with respect to some subsets of attributes. In a more general setting see e.g. [12], [17] tolerance classes are described.

The crisp inclusion of α in β, where $\alpha, \beta \in G_{IS}$ is defined by $\|\alpha\|_{IS} \subseteq \|\beta\|_{IS}$. The non-crisp inclusion, known in KDD, for the case of association rules is defined by means of two thresholds t and t':

$$support_{IS}(\alpha, \beta) = card(\|\alpha \wedge \beta\|_{IS}) \geq t$$

$$accuracy_{IS}(\alpha, \beta) = \frac{support_{IS}(\alpha, \beta)}{card(\|\alpha\|_{IS})} \geq t'.$$

Elementary granule inclusion in a given information system IS can be defined, for example, by

$$\nu_{t,t'}^{IS}(\alpha, \beta) \text{ if and only if } support_{IS}(\alpha, \beta) \geq t \text{ and } accuracy_{IS}(\alpha, \beta) \geq t'.$$

The closeness of granules can be defined by

$$cl_{t,t'}^{IS}(\alpha,\beta) \text{ if and only if } \nu_{t,t'}^{IS}(\alpha,\beta) \text{ and } \nu_{t,t'}^{IS}(\beta,\alpha).$$

Decision rules. One can define inclusion and closeness of granules corresponding to rules of the form **if** α **then** β using accuracy coefficients.

Having granules $g=(\alpha,\beta)$ and $g'=(\alpha',\beta')$ one can define inclusion and closeness of g and g' by $\nu_{t,t'}^{IS}(g,g')$ if and only if $\nu_{t,t'}^{IS}(\alpha,\alpha')$ and $\nu_{t,t'}^{IS}(\beta,\beta')$.

The closeness can be defined by

$$cl_{t,t'}^{IS}(g,g') \text{ if and only if } \nu_{t,t'}^{IS}(g,g') \text{ and } \nu_{t,t'}^{IS}(g',g).$$

Another way of defining inclusion of granules corresponding to decision rules is as follows

$$\nu_{t}^{IS}(g,g') \text{ if and only if } \nu_{t_1,t_2}^{IS}(\alpha,\alpha') \text{ and } \nu_{t_1,t_2}^{IS}(\beta,\beta')$$

and $t=w_1\bullet t_1+w_2\bullet t_2$, where w_1,w_2 are some given weights satisfying $w_1+w_2=1$ and $w_1,w_2\geq 0$.

Extensions of elementary granules by tolerance relation. For extensions of elementary granules defined by tolerance relation, i.e., granules of the form (α,τ), (β,τ) one can consider the following *inclusion measure*:

$\nu_{t,t'}^{IS}((\alpha,\tau),(\beta,\tau))$ if and only if $\nu_{t,t'}^{IS}(\alpha,\beta)$ for any α',β' such that $(\alpha,\alpha')\in\tau$ and $(\beta,\beta')\in\tau$

and the following *closeness measure*:

$cl_{t,t'}^{IS}((\alpha,\tau),(\beta,\tau))$ if and only if $\nu_{t,t'}^{IS}((\alpha,\tau),(\beta,\tau))$ and $\nu_{t,t'}^{IS}((\beta,\tau),(\alpha,\tau),)$.

Sets of rules. An important problem related to association rules is that the number of such rules generated even from simple data table can be large. Hence, one should search for methods of aggregating close association rules. We suggest that this can be defined as searching for some close information granules.

Let us consider two finite sets $Rule\_Set$ and $Rule\_Set'$ of association rules defined by

$$Rule\_Set=\{(\alpha_i,\beta_i):i=1,\ldots,k\},\ Rule\_Set'=\{(\alpha_i',\beta_i'):i=1,\ldots,k'\}.$$

One can treat them as higher order information granules. These new granules $Rule\_Set$, $Rule\_Set'$ can be treated as close in a degree at least t (in IS) if and only if there exists a relation rel between sets of rules $Rule\_Set$ and $Rule\_Set'$ such that:

- For any $Rule$ from the set $Rule\_Set$ there is $Rule'$ from $Rule\_Set'$ such that $(Rule,Rule')\in rel$ and $Rule$ is close to $Rule'$ in degree at least t.
- For any $Rule'$ from the set $Rule\_Set'$ there is $Rule$ from $Rule\_Set$ such that $(Rule,Rule')\in rel$ and $Rule$ is close to $Rule'$ in degree at least t.

Granules defined by sets of granules. The previously discussed methods of inclusion and closeness definition can be easily adopted for the case of granules defined by sets of already defined granules. Let G, H be sets of granules.

The inclusion of G in H can be defined by

$$\nu_{t,t'}^{IS}(G,H) \text{ if and only if for any } g \in G \text{ there is } h \in H \text{ for which } \nu_{t,t'}^{IS}(g,h)$$

and the closeness by

$$cl_{t,t'}^{IS}(G,H) \text{ if and only if } \nu_{t,t'}^{IS}(G,H) \text{ and } \nu_{t,t'}^{IS}(H,G,).$$

Inclusion for complex granules specified by inclusion of their parts is symbolized in Figure 1.

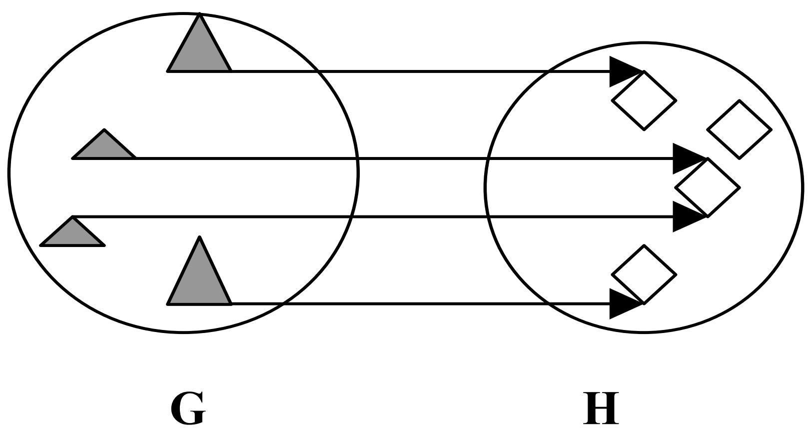

Figure 1. Two Sets of Granules

One can also extend other methods for measuring the inclusion and closeness of granules defined by sets of already defined granules.

Rough Sets in Distributed Systems

In this section we consider operations on approximation spaces which seem to be important for approximate reasoning in distributed systems. We consider a set of agents Ag. Each agent is equipped with some approximation spaces. Agents are cooperating to solve a problem specified by a special agent called *customer-agent*. The result of cooperation is a scheme of agents. In the simplest case the scheme can be represented by a tree labeled by agents. In this tree leaves are delivering some concepts and any non-leaf agent $ag \in Ag$ is performing an operation $o(ag)$ on approximations of concepts delivered by its children. The root agent returns a concept being the result of computation by the scheme on concepts delivered by leaf agents. It is important to note that different agents use different languages. Hence concepts delivered by one agent ag_1 can only be perceived in an approximate sense by another agent ag, for illustration see Figures 2, 3 and 4.

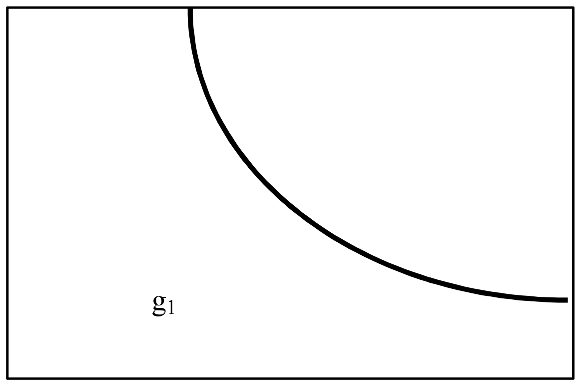

Figure 2. Concept g_1 - Information Granule of $ag_1 \in Ag$

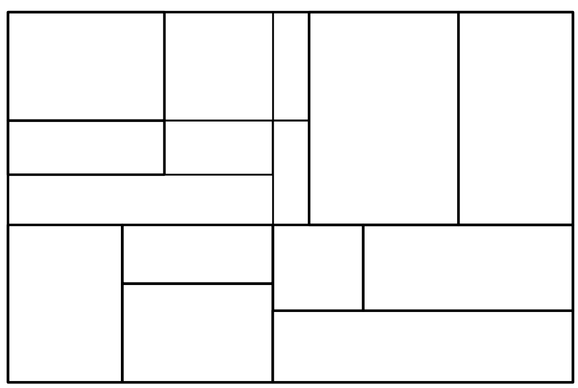

Figure 3. Communication Interface Defined by Data Table

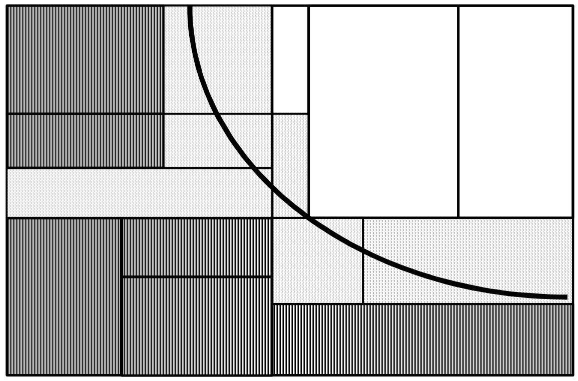

Figure 4. Lower and Upper Approximation of g_1 by $ag \in Ag$

We assume any non leaf-agent $ag \in Ag$ is equipped with an operation

$$o(ag): U_{ag}^{(1)} \times \ldots \times U_{ag}^{(k)} \rightarrow U_{ag}^{(0)}$$

and has different approximation spaces

$$AS_{ag}^{(i)} = \left(U_{ag}^{(i)}, I_{ag}^{(i)}, \nu_{SRI}\right), \text{ where } i = 0, \ldots, k .$$

We assume that the agent ag perceives objects by measuring values of some available attributes. Hence some objects can become indiscernible [6]. This influences the specification of any operation $o(ag)$. We consider a case when attribute value vectors represent arguments and values of operations. Hence instead of the operation $o(ag)$ we have its inexact specification $o^*(ag)$ taking as arguments $I_{ag}^{(1)}(x_1), \ldots, I_{ag}^{(k)}(x_k)$ for some $x_1 \in U_{ag}^{(1)}, \ldots, x_k \in U_{ag}^{(k)}$ and returning the value $I_{ag}^{(0)}(o(ag)(x_1, \ldots, x_k))$ if $o(ag)(x_1, \ldots, x_k)$ is defined, otherwise returning the empty set. This operation can be extended to the operation $o^*(ag)$ with arguments being definable sets (in approximation spaces attached to arguments) and with values in the family of all non-empty subsets of $U_{ag}^{(0)}$. Let $X_1, \ldots, X_k$ be definable sets. We define

$$o^*(ag)(X_1, \ldots, X_k) = \bigcup_{x_1 \in X_1, \ldots, x_k \in X_k} o^*(ag)\left(I_{ag}^{(1)}(x_1), \ldots, I_{ag}^{(k)}(x_k)\right).$$

In the sequel, for simplicity of notation, we write $o(ag)$ instead of $o^*(ag)$.

This idea can be formalized as follows. First we define terms representing schemes of agents.

Let $X_{ag}, Y_{ag}, \ldots$ be agent variables for any leaf-agent $ag \in Ag$. Let $o(ag)$ denote a function of arity k. We have mentioned that it is an operation from Cartesian product of $Def\_Sets\left(AS_{ag}^{(1)}\right)$, ..., $Def\_Sets\left(AS_{ag}^{(k)}\right)$ into $P\left(U_{ag}^{(0)}\right)$, where $Def\_Sets\left(AS_{ag}^{(i)}\right)$ denotes the family of sets definable in $AS_{ag}^{(i)}$. Using the above variables and functors we define terms in a standard way, for example

$$t = o(ag)(X_{ag_1}, X_{ag_2}).$$

Such terms can be treated as description of complex information granules. By a valuation we mean any function val defined on the agent variables with values being definable sets satisfying $val(X_{ag}) \subseteq U_{ag}^{(0)}$ for any leaf-agent $ag \in Ag$. Now we can define the lower and the upper values of any term t under the valuation val with respect to a given approximation space $AS_{ag}^{(i)}$ of an agent ag.

1. If t is of the form X_{ag_i} and $val(t) \subseteq U_{ag}^{(i)}$ then

$$val(LOW, AS_{ag}^{(i)})(t) = LOW(AS_{ag}^{(i)}, val(t))$$

$$val(UPP, AS_{ag}^{(i)})(t) = UPP(AS_{ag}^{(i)}, val(t))$$

else the lower and the upper values are undefined.

2. If $t = o(ag)(t_1, \ldots, t_k)$, where $t_1, \ldots, t_k$ are terms and $o(ag)$ is an operation of arity k, then

a) **if** for $i = 1, \ldots, k$ $val(LOW, AS_{ag}^{(i)})(t_i)$ is defined

then

$$val(LOW, AS_{ag}^{(0)})(t) = LOW(AS_{ag}^{(0)}, o(ag)\,(val(LOW, AS_{ag}^{(1)})(t_1), \ldots, val(LOW, AS_{ag}^{(k)})(t_k)))$$

else $val(LOW, AS_{ag}^{(0)})(t)$ is undefined,

b) **if** for $i = 1, \ldots, k$ $val(UPP, AS_{ag}^{(i)})(t_i)$ is defined

then

$$val(UPP, AS_{ag}^{(0)})(t) = UPP(AS_{ag}^{(0)}, o(ag)\,(val(UPP, AS_{ag}^{(1)})(t_1), \ldots, val(UPP, AS_{ag}^{(k)})(t_k)))$$

else $val(UPP, AS_{ag}^{(0)})(t)$ is undefined.

Example

Let $Ag = \{ag, ag_1, ag_2\}$ be a set of agents and let $cust$ be an agent called customer. We explain how agents from Ag produce a granule defined by a given term and how this granule is related to the concept specified by $cust$. A binary operation $o(ag)$ of ag and an information system

$$IS_{cust} = (\{w_1, w_2, w_3, w_4, w_5, w_6\}, d)$$

are described in Table 4. We assume that objects w_i, where $i = 1, \ldots, 6$ are perceived by ag using a_1^0. Two information systems IS_{ag_1}, IS_{ag_2} presented in Table 1 describe input information granules. Data tables $DT_1 = (U_{ag}^{(1)}, A_{ag}^{(1)} \cup \{d_1\})$ and $DT_2 = (U_{ag}^{(2)}, A_{ag}^{(2)} \cup \{d_2\})$ described in Table 2 and Table 3 characterize communication interfaces between agents ag_1, ag_2 and ag. A binary operation, input information granules and communication interfaces are illustrated in Figure 5. The first four columns of Table 4 define the information system IS_{ag_i} and the approximation space $AS_{ag}^{(i)} = \left(U_{ag}^{(i)}, A_{ag}^{(i)}, \nu_{SRI}\right)$, where $i = 1,2$.

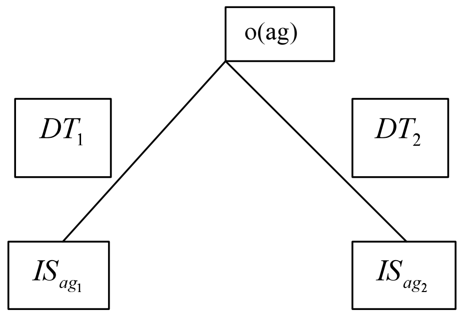

Figure 5. Operation, Input Granules and Communication Interfaces

$U_{ag_1}^{(0)}$	d_1
y_1	1
y_2	0
y_3	1
y_4	0

$U_{ag_2}^{(0)}$	d_2
z_1	1
z_2	1
z_3	1
z_4	0

Table 1 Information Systems IS_{ag_1} and IS_{ag_2}

$U_{ag}^{(1)}$	a_1^1	a_2^1	a_3^1	d_1	$I_{ag}^{(1)}$
y_1	yes	Yes	no	1	$\{y_1\}$
y_2	no	Yes	no	0	$\{y_2, y_3\}$
y_3	no	Yes	no	1	$\{y_2, y_3\}$
y_4	no	No	yes	0	$\{y_4\}$

Table 2 Data Table DT_1 and Uncertainty Function $I_{ag}^{(1)}$

$U_{ag}^{(2)}$	a_1^2	a_2^2	a_3^2	d_2	$I_{ag}^{(2)}$
z_1	yes	Yes	yes	1	$\{z_1, z_2\}$
z_2	yes	Yes	yes	1	$\{z_1, z_2\}$
z_3	no	No	yes	1	$\{z_3, z_4\}$
z_4	no	No	yes	0	$\{z_3, z_4\}$

Table 3 Data Table DT_2 and Uncertainty Function $I_{ag}^{(2)}$

a_1^1	a_2^1	a_3^1	a_1^2	a_2^2	a_3^2	a_1^0	$U_{ag}^{(0)}$	**d**
yes	yes	No	yes	yes	yes	1	w_1	+
yes	yes	No	no	no	yes	2	w_2	+
no	yes	No	yes	yes	yes	3	w_3	+
no	yes	No	no	no	yes	4	w_4	-
no	no	Yes	yes	yes	yes	5	w_5	-
no	no	Yes	no	no	yes	6	w_6	-

Table 4 Operation $o(ag)$ and Customer Information System

For example we compute the lower value of $t = o(ag)(X_{ag_1}, X_{ag_2})$.

Let $val(X_{ag_1}) = \{y_1, y_3\}$ and $val(X_{ag_2}) = \{z_1, z_2, z_3\}$. Hence

$val(LOW, AS_{ag}^{(1)})(X_{ag_1}) = \{y_1\}$ and $val(LOW, AS_{ag}^{(2)})(X_{ag_2}) = \{z_1, z_2\}$.

We obtain the lower value

$val(LOW, AS_{ag}^{(0)})(o(ag)(X_{ag_1}, X_{ag_2})) = \{w_1\}$.

The support of the rule

if t **then** $d = +$

under the valuation val with respect to the lower approximations is equal to 1 and the accuracy is also equal to 1.

On the other hand one can obtain that, the support of the rule **if** t **then** $d = +$ under the valuation val with respect to the upper approximations is equal to 3 and the accuracy is equal to 0.75.

Let us observe that the set $val(UPP, AS_{ag}^{(0)})(t) - val(LOW, AS_{ag}^{(0)})(t)$ can be treated as the boundary region of t under val. Moreover, in the process of term construction we have additional parameters to be tuned for obtaining sufficiently high support and accuracy, namely the approximation operations.

We conclude this section by formulating some examples of basic algorithmic problems.

- *Synthesis of generalized association rules.* Searching for a scheme (term t) over a given set Ag of agents and for a valuation val such that the rule **if** t **then** t', where t' is a concept description specified by customer-agent, has the support at least s and the accuracy at least c under the valuation val.

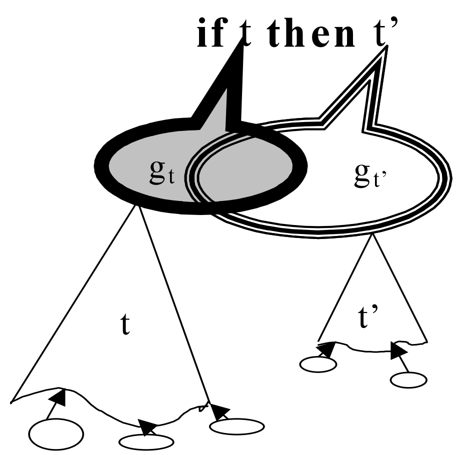

Figure 6. Generalized Association Rules

- *Synthesis of concepts close to the concept specified by the customer-agent.* Searching for a scheme (term t) over a given set Ag of agents and a set Val of valuations such that the concept specified by the customer-agent is sufficiently close to t under Val and the total size of the term t and the set Val is minimal.

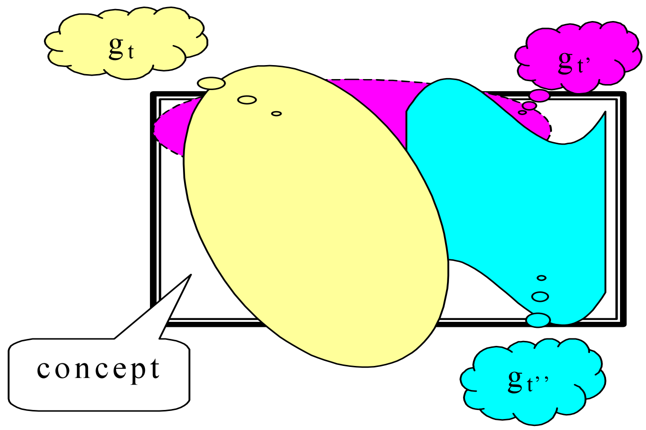

Figure 7. Customer Concept Approximation

Robustness of Granules

Extracting robust patterns (i.e. patterns with properties not deviating to much under small disturbances of parameters) from data is an important task for many applications, in particular related to KDD.

The robustness of constructed granules can be defined by a notion analogous to the continuity of function in a given point.

Definition

Let $f : X_1 \times \ldots \times X_k \to X$ and let us assume closeness relations $cl_p^{(i)}(\bullet,\bullet)$ and $cl_p(\bullet,\bullet)$ in X_i for $i = 1,\ldots,k$ and X are given. We say that f is $(\varepsilon;\varepsilon_1,\ldots,\varepsilon_k)$ - *robust* in a given point $(x_1,\ldots,x_k)$ if and only if for any $(y_1,\ldots,y_k)$ from $X_1 \times \ldots \times X_k$

$$\textbf{if } cl_{1-\varepsilon_i}^{(i)}(x_i, y_i) \text{ for } i = 1,\ldots,k \textbf{ then } cl_{1-\varepsilon}(f(x_1,\ldots,x_k), f(y_1,\ldots,y_k)).$$

We can formulate useful property for robustness checking.

Proposition

Let an operation $F(ag):\prod_{i=1}^{k} Def\_Sets\left(AS_{ag}^{(i)}\right)\rightarrow Def\_Sets\left(AS_{ag}^{(0)}\right)$ be $(\varepsilon;\varepsilon_1,\ldots,\varepsilon_k)$ - robust in $(g_1,\ldots,g_k)$ and let for every agent ag_i operation $F(ag_i):\prod_{j=1}^{l_i} Def\_Sets\left(AS_{ag}^{(j)}\right)\rightarrow Def\_Sets\left(AS_{ag}^{(i)}\right)$ be $\left(\varepsilon_i;\varepsilon_1^{(i)},\ldots,\varepsilon_{l_i}^{(i)}\right)$ - robust in $\left(g_1^{(i)},\ldots,g_{l_i}^{(i)}\right)$ for $i=1,\ldots,k$. Then the superposition of $F(ag)$ and $F(ag_1),\ldots,F(ag_k)$ is $\left(\varepsilon;\varepsilon_1^{(1)},\ldots,\varepsilon_{l_i}^{(1)},\ldots,\varepsilon_1^{(k)},\ldots,\varepsilon_{l_k}^{(k)}\right)$ - robust in $\left(g_1^{(1)},\ldots,g_{l_1}^{(1)},\ldots,g_1^{(k)},\ldots,g_{l_k}^{(k)}\right)$.

In the above Proposition we consider operations with values being definable sets. These operations can be received from $o^*(ag)$ by applying the lower (or upper) approximation operation in $AS_{ag}^{(0)}$ to the values of $o^*(ag)$. One can easily derive the robustness condition for terms describing the construction of information granules.

An important practical problem is to discover rules for decomposition of a given threshold ε into thresholds $\varepsilon_1,\ldots,\varepsilon_k$ for a given operation $F(ag)$ in such a way that $F(ag)$ is $(\varepsilon;\varepsilon_1,\ldots,\varepsilon_k)$ - robust in a given point.

Now, we will discuss applications of closeness between granules for deducing that they share close sets of properties. Informally speaking, close granules should share similar properties. Assuming G is a given set of granules and $R\subseteq G\times G$ is a relation describing similarity, indiscernibility or similar functionality of information granules and $Prop$ is a given set of properties (unary predicates) defined on information granules we can formulate the relationship between the relation R and the set $Prop$ of properties as follows:

$$\forall_{g,g'\in G}\forall_{P\in Prop}\left(R(g,g')\rightarrow\left(P(g)\rightarrow P(g')\right)\right).$$

The property is called $(R,Prop)$ - *relationship*. Searching methods for relation R satisfying the specified above condition is an important problem.

In case of multi-agent environment one should consider different $(R,Prop)$ -relationships for different agents. One of the fundamental question is how to deduce that close complex information granules have similar properties.

The main role in such deduction play, so called *decomposition rules*. They have the following form:

$$o(ag): \frac{P_{ag}(\bullet), \varepsilon, st(ag), cl^{ag}_{1-\varepsilon}(\bullet,\bullet)}{P^{(1)}_{ag_1}(\bullet), \varepsilon_1, st(ag_1), cl^{ag_1}_{1-\varepsilon_1}(\bullet,\bullet); \ldots; P^{(k)}_{ag_k}(\bullet), \varepsilon_k, st(ag_k), cl^{ag_k}_{1-\varepsilon_k}(\bullet,\bullet)},$$

where $o(ag)$ is k - ary operation of ag; $st(ag)$, $st(ag_1)$, ..., $st(ag_k)$ are standard information granules (prototypes); $P_{ag}(\bullet)$, $P^{(1)}_{ag_1}(\bullet)$, ..., $P^{(k)}_{ag_k}(\bullet)$ are properties; $\varepsilon, \varepsilon_1, \ldots, \varepsilon_k$ are uncertainty coefficients; $cl^{ag}_{1-\varepsilon}$, $cl^{ag_1}_{1-\varepsilon_1}$, ..., $cl^{ag_k}_{1-\varepsilon_k}$ are closeness relations attached to agents ag, ag_1, ..., ag_k, respectively.

We assume the decomposition rule is true if the following conditions are satisfied:

- $o(ag)(st(ag_1), \ldots, st(ag_k)) = st(ag)$,
- $P^{(1)}_{ag_1}(st(ag_1)) \wedge \ldots \wedge P^{(k)}_{ag_k}(st(ag_k)) \rightarrow P_{ag}(st(ag))$
- $cl^{ag_1}_{1-\varepsilon_1}(st(ag_1), g_1) \wedge \ldots \wedge cl^{ag_k}_{1-\varepsilon_k}(st(ag_k), g_k) \rightarrow cl^{ag}_{1-\varepsilon}(st(ag), g)$ for any $g_1, \ldots, g_k \in G$, where $g = o(ag)(g_1, \ldots, g_k)$,
- $cl^{ag}_{1-\varepsilon}(g, g') \rightarrow (P_{ag}(g) \rightarrow P_{ag}(g'))$ for any $g, g' \in G$.

The first condition states that the standard granule at ag is equal to the result of operation $o(ag)$ on standard granules $st(ag_1)$, ..., $st(ag_k)$. The second condition allows to deduce that the standard granule $st(ag)$ has property P_{ag} if the standard granules $st(ag_1)$, ..., $st(ag_k)$ have properties $P^{(1)}_{ag_1}$, ..., $P^{(k)}_{ag_k}$, respectively. The next condition allows to infer that $st(ag)$ is close to $o(ag)(g_1, \ldots, g_k)$ in degree at least $1-\varepsilon$ if the standard granule $st(ag_i)$ is close to g_i in degree at least $1-\varepsilon_i$ for any $i = 1, \ldots, k$. The last condition means that if granules g, g' are close in degree at least $1-\varepsilon$ and g has property P_{ag} then g' has also this property.

Hence we obtain the following basic lemma:

Lemma

Assuming the decomposition rule for $o(ag)$ is true and

$$P^{(1)}_{ag_1}(st(ag_1)), cl^{ag_1}_{1-\varepsilon_1}(st(ag_1), g_1), \ldots, P^{(k)}_{ag_k}(st(ag_k)), cl^{ag_k}_{1-\varepsilon_k}(st(ag_k), g_k) \text{ hold}$$

we obtain that

$$P_{ag}(g) \text{ holds, where } g = o(ag)(g_1, \ldots, g_k).$$

One of the challenges for knowledge discovery is to discover decomposition rules for operations possessed by agents.
It is easy to observe that by repeated application of decomposition rules one can derive sufficient conditions for synthesis of robust complex information granules.

Conclusions

Our approach can be treated as a step towards understanding of complex information granules and their role in spatial and temporal reasoning. We have discussed information granule syntax and semantics as well as their inclusion and closeness. Several examples of information granules have been presented. Methods for synthesis of complex information granules in distributed environment are important for spatial and temporal reasoning because of existing need for approximate fusion (merging) of information from different sources of spatial and temporal information. An approach for synthesis and analysis of complex information granules in distributed environment has been discussed. It was emphasized the necessity of robust information granules synthesis. The robustness guarantees that a properly constructed scheme of reasoning on prototypes (standards) leads to conclusion satisfying a given specification also when the standards are substituted by sufficiently small deviations of them.
There is still much work to be done to create foundations for spatial and temporal reasoning. Among the issues to be investigated are methods for synthesis of terms for large families of tasks, including among other problems of

- discovery of decomposition rules,
- synthesis from data rules for uncertainty coefficient propagation,
- representation of discovered temporary schemes of reasoning,
- efficient identification of relevant fragments of synthesized
- schemes of reasoning to be used in further solution synthesis,
- fusion operation discovery,
- adaptive spatial and temporal reasoning.

Many of these problems are closely related to Knowledge Discovery and Data Mining goals [11].

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